2.6-Limits at Infinity

In 2.2, we learned that if $y \to \pm\infty$ as $x \to a$, then the graph of f has a *vertical asymptote* at $x = a$. Similarly, if $y \to L$ as $x \to \pm\infty$, then the graph of the function has a *horizontal asymptote* at $y = L$.

Key Limit: $\lim_{x\to\infty} \frac{1}{x} = .$ 0 $\qquad \lim_{x\to-\infty} \frac{1}{x} = 0$

Computing limits at Infinity:

1. Work on fractional expressions
2. Factor out highest power (num/denom)
   "dominating term"
3. Cancel and use key limits

Examples:

$$\lim_{x\to\infty} \frac{5x^2+7}{3x^2-x}$$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}\left(5+\frac{7}{x^2}\right)}{\cancel{x^2}\left(3-\frac{1}{x}\right)} \qquad \frac{7}{x^2}\to 0,\quad \frac{1}{x}\to 0,\quad 7\left(\frac{1}{x}\right)^2$$

$$= \boxed{\frac{5}{3}}$$

$$\lim_{x\to-\infty} \frac{x-2}{x^2+2x+1}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x}\left(1-\frac{2}{x}\right)}{x^{\cancel{2}}\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{1}{x}\cdot\frac{1-\frac{2}{x}}{1+\frac{2}{x}+\frac{1}{x^2}} \qquad \frac{1}{x}\to 0,\quad \frac{2}{x}\to 0,\quad \frac{1}{x^2}\to 0$$

$$= 0\cdot 1$$

$$= \boxed{0}$$

Compute $\lim\limits_{x\to\infty}\dfrac{\left(\sqrt{x^2+5x}-x\right)}{1}\cdot\dfrac{\left(\sqrt{x^2+5x}+x\right)}{\left(\sqrt{x^2+5x}+x\right)}$

$$= \lim_{x\to\infty}\frac{\cancel{x^2}+5x-\cancel{x^2}}{\sqrt{x^2+5x}+x}$$

$$= \lim_{x\to\infty}\frac{\cancel{x}(5)}{\cancel{x}\left(\sqrt{1+\underbrace{\tfrac{5}{x}}_{0}}+1\right)}$$

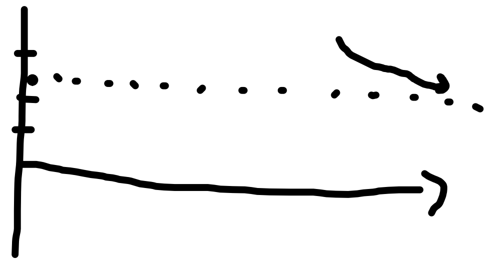
$$= \boxed{\frac{5}{2}}$$

$$\sqrt{12}=\sqrt{4\cdot 3}=2\sqrt{3}$$

$$\sqrt{x^2+5x}$$

$$=\sqrt{x^2\left(1+\frac{5}{x}\right)}$$

$$= x\sqrt{1+\frac{5}{x}}$$



Find the horizontal asymptotes of $f(x) = \dfrac{\sqrt{x^2+2}}{3x-6}$.

$\sqrt{x^2} = |x|$

(Reminder: V.A. $x=2$ $\quad \frac{\text{Nonzero}}{0}$)

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\sqrt{1+\frac{2}{x^2}}}{\cancel{x}\left(3-\frac{6}{x}\right)}$$

$\frac{2}{x^2} \to 0$, $\frac{6}{x} \to 0$

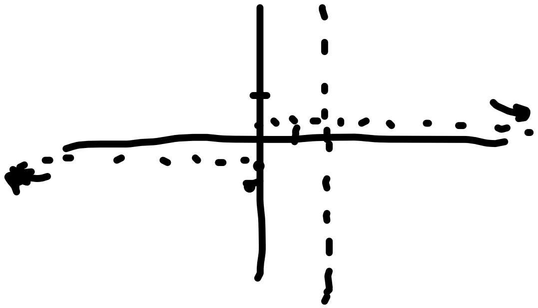
$$= \boxed{\frac{1}{3}}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x\to-\infty} \frac{-\cancel{x}\sqrt{1+\frac{2}{x^2}}}{\cancel{x}\left(3-\frac{6}{x}\right)}$$

$\frac{2}{x^2} \to 0$, $\frac{6}{x} \to 0$

$$= \boxed{-\frac{1}{3}}$$



Behavior of Polynomials:

As stated in class, as $x \to \infty$, $x^n \to \infty$.

As $x \to -\infty$, $x^n \to \infty$ if n even, $-\infty$ if n odd

Look at dominating term (highest power)

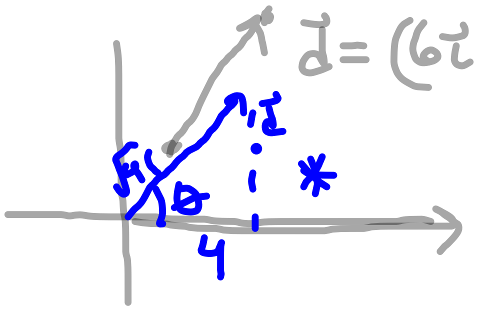
1. degree - odd or even
2. coefficient - ~~+~~ or -

Ex $\lim_{x \to -\infty} \underbrace{-2}_{\text{neg}} x^{\overset{\text{odd}}{3}} + 3x^2 - 1000 = \boxed{+\infty}$ $\left(\text{"}-2 \cdot -\infty\text{"}\right)$

No calculator, no notes!

A force of 20 N acts in the positive *x* axis direction and moves an object from the point (2, 3) to the point (6, 8) (in meters). Find the work done.

Answer : 80 J



$\vec{d} = (6\vec{i}+8\vec{j}) - (2\vec{i}+3\vec{j}) = 4\vec{i}+5\vec{j}$

$F = 20\vec{i}$

$W = \vec{F}\cdot\vec{d}$

$= (20)(4) + (0)(5)$

$= \boxed{80 \text{ N·m or } 80 \text{ J}}$

Alternative

$|\vec{F}| = 20$

$|\vec{d}| = \sqrt{41}$

$\cos\theta = \frac{4}{\sqrt{41}}$ *

$W = |\vec{F}||\vec{d}|\cos\theta$

$= 20\sqrt{41}\cdot\frac{4}{\sqrt{41}} = \boxed{80 \text{ J}}$