$\frac{\text{nonzero}}{0}$

2.6-Limits at Infinity

In 2.2, we learned that if $y \to \pm\infty$ as $x \to a$, then the graph of f has a *vertical asymptote* at $x = a$. Similarly, if $y \to L$ as $x \to \pm\infty$, then the graph of the function has a *horizontal asymptote* at $y = L$.

Key Limit: $\lim_{x\to\infty} \frac{1}{x} =. 0$ $\quad\quad \lim_{x\to-\infty} \frac{1}{x} = 0$

Computing limits at Infinity:

1. only applies with fractional expressions
2. factor out highest power of num/denom "dominating term"
3. cancel and use key limits above

Examples:

$$\lim_{x\to\infty} \frac{5x^2+7}{3x^2-x}$$

$7\left(\frac{1}{x}\right)^2$ $\to 0$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}\left(5+\frac{7}{x^2}\right)}{\cancel{x^2}\left(3-\frac{1}{x}\right)}$$

$\frac{7}{x^2} \to 0$, $\frac{1}{x} \to 0$

$$= \boxed{\frac{5}{3}}$$

$$\lim_{x\to-\infty} \frac{x-2}{x^2+2x+1}$$

$$= \lim_{x\to-\infty} \frac{\cancel{x}\left(1-\frac{2}{x}\right)}{x^{\cancel{2}}\left(1+\frac{2}{x}+\frac{1}{x^2}\right)}$$

$$= \lim_{x\to-\infty} \frac{1}{x}\cdot\frac{1-\frac{2}{x}}{1+\frac{2}{x}+\frac{1}{x^2}}$$

$\frac{1}{x} \to 0$, $\frac{2}{x} \to 0$, $\frac{2}{x} \to 0$, $\frac{1}{x^2} \to 0$

$$= \quad 0\cdot 1 = \boxed{0}$$

Compute $\lim_{x\to\infty} \frac{\sqrt{x^2+5x}-x}{1} \cdot \frac{(\sqrt{x^2+5x}+x)}{(\sqrt{x^2+5x}+x)}$

$$= \lim_{x\to\infty} \frac{\cancel{x^2}+5x-\cancel{x^2}}{\sqrt{x^2+5x}+x}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}(5)}{\cancel{x}\left(\sqrt{1+\frac{5}{x}}+1\right)}$$

$\frac{5}{x}\to 0$

$$= \frac{5}{\sqrt{1}+1} = \boxed{\frac{5}{2}}$$

Recall:

$$\sqrt{12} = \sqrt{4\cdot 3} = 2\sqrt{3}$$

Note:

$$\sqrt{x^2+5x}$$

$$= \sqrt{x^2\left(1+\frac{5}{x}\right)}$$

$$= x\sqrt{1+\frac{5}{x}}$$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Find the horizontal asymptotes of $f(x) = \dfrac{\sqrt{x^2+2}}{3x-6}$.

(Recall: VA $x=2$ $\frac{\text{nonzero}}{0}$)

$\sqrt{x^2} = |x| = -x$ when $x<0$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\sqrt{1+\frac{2}{x^2}}}{\cancel{x}\left(3-\frac{6}{x}\right)}$$

$\frac{2}{x^2} \to 0$, $\frac{6}{x} \to 0$

$$= \boxed{\frac{1}{3}}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+2}}{3x-6}$$

$$= \lim_{x\to-\infty} \frac{-\cancel{x}\sqrt{1+\frac{2}{x^2}}}{\cancel{x}\left(3-\frac{6}{x}\right)}$$

$\frac{2}{x^2} \to 0$, $\frac{6}{x} \to 0$

$$= \boxed{-\frac{1}{3}}$$

Behavior of Polynomials:

As stated in class, as $x \to \infty$, $x^n \to \infty$.

As $x \to -\infty$, $x^n \to \infty$ if n even, $-\infty$ if n odd

Look at dominating term (highest power)

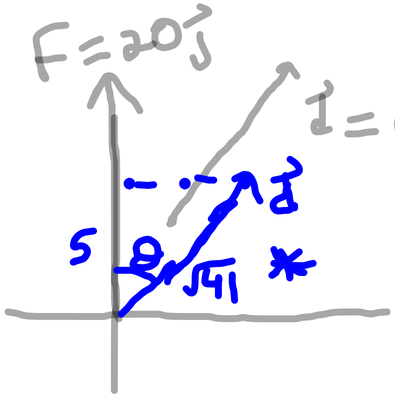
1. degree – odd or even
2. coefficient – $+$ or $-$

Ex $\lim_{x \to -\infty} \underset{\text{neg}}{-2}x^{3}{}^{\text{odd}} + 3x^2 - 1000 = \boxed{+\infty}$ ("$-2 \cdot -\infty$")

Lecture Quiz #1

A force of 20 Newtons acting in the positive ~~x~~ y axis moves an object from the point (2, 3) to the point (6, 8). Find the work done by the force.

(meters)



100 J

$\vec{d} = (6\vec{i}+8\vec{j}) - (2\vec{i}+3\vec{j}) = 4\vec{i}+5\vec{j}$

$W = \vec{F}\cdot\vec{d}$

$= (0)(4)+(20)(5)$

$= \boxed{100 \text{ N·m or } 100\text{ J}}$

Alternative:

$|\vec{F}| = 20$

$|\vec{d}| = \sqrt{41}$

$\cos\theta = \frac{5}{\sqrt{41}}$ *

$W = |\vec{F}||\vec{d}|\cos\theta$

$= 20\,\cancel{\sqrt{41}}\cdot\frac{5}{\cancel{\sqrt{41}}} = \boxed{100\text{ J}}$