2.7: Tangents, Velocities, and Rates of Change

We are now ready to find a formal way of computing the slope of a line tangent to a curve at a point. Re-view the animation from 2.1 posted on my webpage. What happens as the second x coordinate moves closer to the given tangent-line point?

Each secant line above passes through the point (1, 1). If x is the x-coordinate of the second point, write an expression for the slope of the line between the two points.

Write and solve a limit problem which allows us to find the slope of the tangent line at x = 1.

**More General:** Draw any function, a tangent line, and a secant line in the space below. Label the tangent-line point \((a, f(a))\) and the second point on the secant line \((x, f(x))\).

\[
m_{sec} =
\]

What should happen as the point \((x, f(x))\) moves closer to the tangent-line point \((a, f(a))\)? Write a limit which explains this mathematically:

\[
m_{tan} =
\]
**Most General:** Draw any function, a tangent line, and a secant line again in the space below. Label the tangent-line point \((a, f(a))\) and let \(h\) be the distance between the \(x\) values on the secant line. View the new animation posted under today’s notes for a visual understanding of this).

\[m_{\text{sec}} =\]

\[m_{\text{tan}} =\]

The *derivative* of a function at \(x = a\) is given by

**Examples:** Use a limit definition to find the equation of the line tangent to the curve \(f(x) = x^2 + x\) at the point where \(x = 2\).
Use a limit definition to find the derivative of the function \( f(x) = \frac{1}{2x + 1} \). Compute the slopes of the lines tangent to this graph at \( x = 0, x = 1, \) and \( x = -2 \).

**Secant and Tangent Vectors—an Introduction**

Illustration of Velocity Vectors (Secant and Tangent):
**Example:** Find a vector tangent to the curve \( \mathbf{r}(t) = (3t^2 + 4)i + \sqrt{t}j \) at the point \((7,1)\). Then find parametric equations of the line tangent to the curve at this point.