3.11-Linear Approximation and Differentials

**Purpose:** To understand differentials and linear approximations to a function near a certain point.

**Definitions:** Given \( y = f(x) \), the differential \( dx \) represents an independent quantity (a small change in \( x \)). Then the differential \( dy \) is given by:

**Seemingly Unrelated Topic:** Recall graphing \( y = \sin x \). What happened as you zoom in on the point corresponding to \( x = 0 \)?

**Idea:** The tangent line approximates the curve \( y = f(x) \) near \( x = a \).

What is the equation of the line tangent to \( y = f(x) \) at the point where \( x = a \)?

**Definition:** The *Linear Approximation* (or *Linearization*) of \( f \) at \( x = a \) is

**The Connection:**

**Examples:**

Given \( y = \sqrt{x} \), find \( \Delta y \) and \( dy \) if \( x = 4 \) and \( \Delta x = dx = 1 \).
Use differentials to approximate $\cos 62^\circ$

Find the linear approximation of $f(x) = \sqrt{x}$ at $x = \frac{9}{4}$ and use it to approximate $\sqrt{2}$.

The circumference around the middle of a sphere is measured to be 40 cm, with a possible error of $\pm 1$ cm. Use differentials to estimate the possible error in the volume of the sphere.

**On Your Own:** 3.11 #7,10,11,12,14,19,25-30,31,33