3.11-Linear Approximation and Differentials

Purpose: To understand differentials and linear approximations to a function near a certain point.

Definitions: Given $y = f(x)$, the *differential* dx represents an independent quantity (a small change in x). Then the *differential* dy is given by:

Seemingly Unrelated Topic: Recall graphing $y = \sin x$. What happened as you zoom in on the point corresponding to $x = 0$?

Idea: The tangent line approximates the curve $y = f(x)$ near $x = a$.

What is the equation of the line tangent to $y = f(x)$ at the point where $x = a$?

Definition: The *Linear Approximation* (or *Linearization*) of f at $x = a$ is

The Connection:

Examples:

Given $y = \sqrt{x}$, find Δy and dy if $x = 4$ and $\Delta x = dx = 1$.

Use differentials to approximate $\cos 62°$

Find the linear approximation of $f(x) = \sqrt{x}$ at $x = \dfrac{9}{4}$ and use it to approximate $\sqrt{2}$.

The circumference around the middle of a sphere is measured to be 40 cm, with a possible error of ± 1 cm. Use differentials to estimate the possible error in the volume of the sphere.

On Your Own: 3.11 #7,10,11,12,14,19,25-30,31,33