3.12-Newton's Method

Recall: In 2.5, we introduced the Bisection Method for solving equations. This method is long and tedious. Here we introduce another method for solving equations.

The *Linear Approximation* of f at x_0 is given by $L(x) = f(a) + f'(a)(x-a)$

Set this equal to 0 and solve for x. What do you obtain?

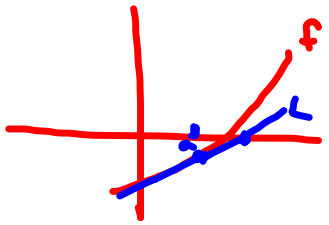
$$L(x) = 0 = f(a) + f'(a)(x-a)$$

$$-f(a) = f'(a)(x-a)$$

$$\frac{-f(a)}{f'(a)} = \frac{f'(a)(x-a)}{f'(a)}$$

$$x - a = \frac{-f(a)}{f'(a)}$$

$$x = a - \frac{f(a)}{f'(a)}$$



Newton's Method

To solve $f(x) = 0$ given a starting value x_0, create a sequence $\{x_1, x_2, x_3 \ldots\}$ using

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

which converge to the solution.

Example: Given $x_0 = 1$ is an approximate solution of $x^2 = 2$, find the solution to 4 decimal places using Newton's Method.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x^2 - 2 = 0$

$f(x) = x^2 - 2$

$f'(x) = 2x$

$x_0 = 1$

$x_1 = 1 - \frac{f(1)}{f'(1)}$ $1^2 - 2 = -1$, $2 \cdot 1 = 2$

$= 1 + \frac{1}{2} = 1.5$ *On quiz/exam, that's all you have to do

$x_2 = 1.5 - \frac{f(1.5)}{f'(1.5)}$ 0.25, 3

≈ 1.4166667

$x_3 = 1.4166667 - \frac{f(-)}{f'(-)} \approx 1.414216$

$x_4 = 1.414216 - \frac{f(-)}{f'(-)} \approx \boxed{1.414214}$

compare to previous answer

(Note: Compare with Bisection Method on $x^2=2$ below)

$1 < x < 2$ $f(1.5) = 2.25$

$1 < x < 1.5$ $f(1.25) = 1.5625$

$1.25 < x < 1.5$ $f(1.375) = 1.890625$

$1.375 < x < 1.5$ $f(1.4375) \approx 2.0664$

$1.375 < x < 1.4375$ $f(1.40625) \approx 1.9775$

$1.40625 < x < 1.4375$ $f(1.421875) \approx 2.021$

$1.40625 < x < 1.421875$ $f(1.4140625) \approx 1.9995$

$1.4140625 < x < 1.421875$ $f(1.41796875) \approx 2.0106$

$1.4140625 < x < 1.41796875$ $f(1.41601562 5) \approx 2.005$

$1.4140625 < x < 1.4160156 25$ $f(1.4150390625) \approx 2.0023$

$1.4140625 < x < 1.4150390625$ $f(1.4145507812 5) \approx 2.00075$

$1.4140625 < x < 1.41455078125$

$x \approx 1.41$ (still only 2 decimal place accuracy!)