$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

3.2-Derivative Rules

Derivative Rules:

If f and g are differentiable functions, then...

$$\frac{d}{dx}(x^n) = n x^{n-1}$$

Power Rule

$$\frac{d}{dx}(c) = 0$$

$$\frac{d}{dx}(cf(x)) = c f'(x)$$

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$$

Ex $f(x) = x^3 - 3x^2$

$$f'(x) = \frac{d}{dx}(x^3) - 3\frac{d}{dx}(x^2)$$

$$= 3x^2 - 3(2x)$$

$$= 3x^2 - 6x$$

$$\frac{d}{dx}(f(x)\cdot g(x)) = \cancel{f'(x)g'(x)}$$

Product Rule

$$f(x)g'(x) + g(x)f'(x)$$

$$\frac{d}{dx}\left(\frac{f(x)^{\text{HI}}}{g(x)_{\text{LO}}}\right) = \cancel{\frac{f'(x)}{g'(x)}}$$

Quotient Rule

LO dHI − HI dLO

$$\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

LO LO

Examples:

Compute the derivative of the following:

$$f(x) = (4x^2 - 1)(7x^3 + x)$$

Method I: Product Rule

$$f'(x) = \boxed{(4x^2-1)(21x^2+1) + (7x^3+x)(8x)}$$

simplify

$$= (84x^4 + 4x^2 - 21x^2 - 1) + (56x^4 + 8x^2)$$

$$= \boxed{140x^4 - 9x^2 - 1}$$

Method II: Multiply first, then differentiate

$$f(x) = 28x^5 + 4x^3 - 7x^3 - x$$

$$f(x) = 28x^5 - 3x^3 - x$$

$$\boxed{f'(x) = 140x^4 - 9x^2 - 1}$$

$$y = \frac{x^2-1}{x^4+1} \quad \begin{matrix}\text{HI}\\ \text{LO}\end{matrix}$$

$$y' = \boxed{\frac{(x^4+1)(2x) - (x^2-1)(4x^3)}{(x^4+1)^2}}$$

Simplify

$$= \frac{(2x^5+2x) + (-4x^5+4x^3)}{(x^4+1)^2}$$

$$= \boxed{\frac{-2x^5+4x^3+2x}{(x^4+1)^2}}$$

(at $x=4$)

Given $f(4) = 3$ and $f'(4) = -5$, find the derivative of:

$g(x) = \sqrt{x} f(x)$ $x^{\frac{1}{2}}$ Product Rule

$g'(x) = \sqrt{x} \cdot f'(x) + f(x) \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$g'(4) = \sqrt{4} \cdot f'(4) + f(4) \cdot \frac{1}{2}(4)^{-1/2}$

$= (2)(-5) + (3) \cdot \frac{1}{2} \cdot \frac{1}{2}$

$= -10 + \frac{3}{4} = \boxed{\frac{-37}{4}}$

$h(x) = \frac{f(x)}{x}$ HI LO Quotient Rule

$h'(x) = \frac{x \cdot f'(x) - f(x) \cdot 1}{x^2}$

$h'(4) = \frac{4 \cdot f'(4) - f(4) \cdot 1}{4^2}$ -5 3

$= \frac{-20 - 3}{16} = \boxed{\frac{-23}{16}}$

Alternate:

$h(x) = f(x) \cdot x^{-1}$ Product Rule

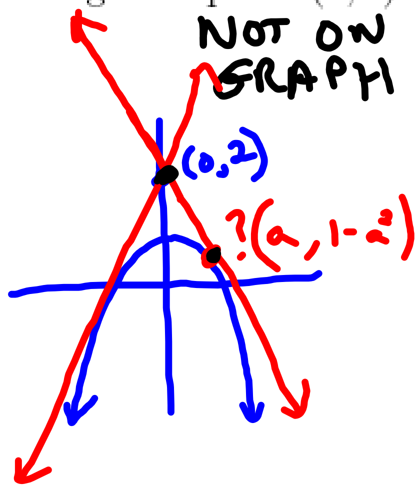
NOTE $\frac{d}{dx}(x^{-1}) = -1x^{-2}$

$f(x)=$
Find the slope of the line(s) tangent to the curve $y = 1 - x^2$ which pass through the point $(0, 2)$.

$$\cancel{f'(x) = -2x \qquad m_{tan} = 2(0) = 0}$$

Let a be the x-coordinate of the tangent-line point

y-coordinate : $1-a^2$



$$f'(x) = -2x$$

$$m_{tan} = f'(a) = -2a$$

$$m_{tan} = \frac{\Delta y}{\Delta x} = \frac{(1-a^2)-2}{a-0}$$

Same line, so equate them

$$a\left(-2a = \frac{-1-a^2}{a}\right)$$

$$-2a^2 = -1 - a^2$$
$$+2a^2 \qquad +2a^2$$

$$0 = a^2 - 1$$
$$0 = (a+1)(a-1)$$
$$a = \pm 1$$

$\underline{x=1}$: $m_{tan} = -2(1) = \boxed{-2}$

$\underline{x=-1}$: $m_{tan} = -2(-1) = \boxed{2}$

Determine whether the function below is differentiable at $x = 1$:

$$f(x) = \begin{cases} x^2 & \text{if } x \leq 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x & \text{if } x < 1 \\ 2 & \text{if } x > 1 \end{cases}$$

$x = 1$:

slopes from left $\rightarrow 2(1) = 2$

slopes from right $\rightarrow 2$

$\therefore f'(1) = 2$

Thus: $f'(1)$ DNE?

f cts at $x = 1$?

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 = 1$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2x+1 = 3$

$\therefore$ lim DNE, so f is not cts at $x=1$,

so f not diff at $x=1$.

3.3 and Bonus PHYS mat'l

$\frac{d}{dt}(\text{position}) = \text{velocity}$

$\frac{d}{dt}(\text{velocity}) = \text{acceleration}$

Antiderivatives (Integrals) → given deriv, find original

Rules:

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$$

$$\int c f(x)\,dx = c\int f(x)\,dx$$

$$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

Antiderivatives (Integrals) → given deriv, find original

Rules:

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Ex A projectile is launched with initial speed v_0 (at angle θ) from the origin. Find equations for the position over time.

$\int a_x = \int 0$

$v_x = C$

init $v_x = v_0 \cos\theta$

$\int v_x\,dt = \int v_0\cos\theta\,dt$

$r_x = (v_0\cos\theta)t + C$

init $r_x = 0$

$$\boxed{r_x = (v_0\cos\theta)t}$$

$\int a_y\,dt = -g\,dt$

initial $v_y = v_0\sin\theta$

$v_y = -gt + C$

$v_0\sin\theta = -g(0) + C \quad C = v_0\sin\theta$

$\int v_y(t)\,dt = \int -gt' + v_0\sin\theta\,dt$

$r_y = -\frac{1}{2}gt^2 + (v_0\sin\theta)t + C$

initial $r_y = 0$

$c = 0$

$$\boxed{r_y = -\frac{1}{2}gt^2 + (v_0\sin\theta)t}$$