3.4: Limits and Derivatives of Trig Functions

\((NOTE:\) we will assume without proof that the functions \(f(x) = \sin x\) and \(g(x) = \cos x\) are continuous.)

**Key Limit:** \(\lim_{x \to 0} \frac{\sin x}{x} = \)

"Proof":

\[
\begin{align*}
\text{Key Limit: } & \lim_{x \to 0} \frac{\cos x - 1}{x} = \\
\text{Proof:}
\end{align*}
\]

We can use these limits to find the derivative of \(f(x) = \sin x\) using the definition:
Similarly, we can show that \( \frac{d}{dx}(\cos x) = \)

Once we know these, we can find the derivatives of all the other trig functions using quotient rules:

Example: \( \frac{d}{dx}(\tan x) = \)

**Summary:** Key limits:

Key derivatives:

**Examples:**

\[
\lim_{x \to 0} \frac{\tan 7x}{\sin 3x}
\]
Differentiate: a) \( f(x) = x^2 \tan x \) 

b) \( y = \frac{1 - \cos x}{\sin x} \)

Find all \( a \in [0, 2\pi] \) such that the line tangent to \( f(x) = \sin^2 x + \cos x \) at \( x = a \) is horizontal.

You already know the identity \( \sin(2x) = 2 \sin x \cos x \). What do you obtain when you differentiate the right hand side of this identity?

**On Your Own:** 3.4 #6,7,8,10,14,17,21,24,32,37,40,44,55