3.4: Limits and Derivatives of Trig Functions

(*NOTE*: we will assume without proof that the functions $f(x) = \sin x$ and $g(x) = \cos x$ are continuous.)

Key Limit: $\lim_{x \to 0} \dfrac{\sin x}{x} =$

"Proof":

Key Limit: $\lim_{x \to 0} \dfrac{\cos x - 1}{x} =$

Proof:

We can use these limits to find the derivative of $f(x) = \sin x$ using the definition:

Similarly, we can show that $\frac{d}{dx}(\cos x) =$

Once we know these, we can find the derivatives of all the other trig functions using quotient rules:

Example: $\frac{d}{dx}(\tan x) =$

Summary: Key limits:

Key derivatives:

Examples:

$$\lim_{x \to 0} \frac{\tan 7x}{\sin 3x}$$

Differentiate: a) $f(x) = x^2 \tan x$ b) $y = \dfrac{1 - \cos x}{\sin x}$.

Find all $a \in [0, 2\pi]$ such that the line tangent to $f(x) = \sin^2 x + \cos x$ at $x = a$ is horizontal.

You already know the identity $\sin(2x) = 2 \sin x \cos x$. What do you obtain when you differentiate the right hand side of this identity?

On Your Own: 3.4 #6,7,8,10,14,17,21,24,32,37,40,44,55