

3.5: The Chain Rule

We know $\frac{d}{dx}(x^2) = 2x$. Does $\frac{d}{dx}((x^2 + 4)^2) = 2(x^2 + 4)$?
(HINT: Expand the function, then differentiate).

Recall: The *composition* of 2 functions f and g is defined by

Define f and g for the above function.

The Chain Rule: If f and g are differentiable functions, $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} =$$

An alternate version of the Chain Rule states that $\frac{d}{dx} f(g(x)) =$

Examples:

Find the derivatives of the following:

$$f(x) = 4 \cos(x^3)$$

$$y = (x^2 - x + 1)^{23}$$

Differentiate the following:

$$f(x) = x^3 \sin^2 x$$

$$y = (1 + x^5 \cot x)^{-8}$$

$$g(x) = \frac{(2x + 3)^3}{(4x^2 - 1)^8}$$

$$y = \left(\frac{x - 5}{2x + 1} \right)^3$$

Given the table of values below, find the derivative of $f(g(x))$ and $g(f(x))$

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
-1	2	3	2	-3
2	0	4	1	-5