3.6-Implicit Differentiation

The equation $F(x, y) = 0$ *implicitly* defines a relation (not necessarily a function) between y and x. The *graph* of $F(x, y) = 0$ is the set of all points (x, y) such that the equation holds ($\{(x, y)|F(x, y) = 0\}$). Given a graph of an implicitly-defined relation, we can still talk about the slope of the line tangent to the curve at a given point.

Method for Implicit Differentiation:

1. Done when y is not explicitly defined as a function of x.
2. Differentiate both sides of the equation, remembering that y depends on x (can call it $y(x)$)
3. Solve for $y'(x)$

Examples:

Find $\dfrac{dy}{dx}$ implicity if $xy = 1$. Then solve for y and show you get the same answer.

Find $\dfrac{dy}{dx}$ if $5y^2 + \sin y = x^2$

Find the slope of the line tangent to $y^2 - x + 1 = 0$ at the point $(2, -1)$

Show that the curves $y = 3x^2$ and $x^2 + 2y^2 = 19$ are orthogonal.

The equations $x^2 + y^2 = r^2$ and $y = mx$ represent *families of curves* for different constants r and m. Show that these families of curves are orthogonal.

On Your Own: 3.6 #5,7,9,13,21,22,33,35,36,37,39,41,45