3.8-Higher Derivatives

Examples:

Find the first and second derivatives of the following functions:

a) $f(x) = \dfrac{1}{x^2+1} = (x^2+1)^{-1}$

$f'(x) = -1(x^2+1)^{-2}(2x) = \dfrac{-2x}{(x^2+1)^2}$

$f''(x) = 2(x^2+1)^{-3}(2x)(2x) + -(x^2+1)^{-2}(2)$ simplify

$\dfrac{(x^2+1)^2}{(x^2+1)^{-3}}$

$= 2(x^3+1)^{-3}[4x^2 - (x^2+1)(2)]$

$= \boxed{2(x^3+1)^{-3}(2x^2-1)}$

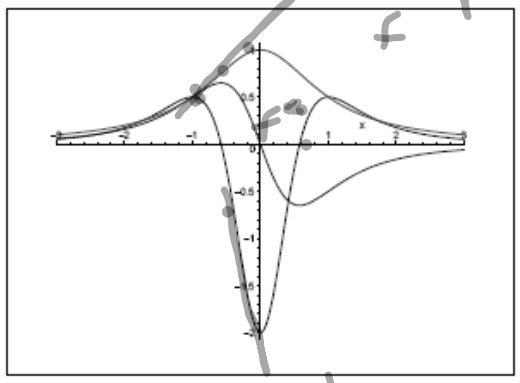
b) $y = \sin^2 x$ $(\sin x)^2$

$\dfrac{dy}{dx} = 2\sin x \cos x$

$= \sin(2x)$

$\dfrac{d^2y}{dx^2} = \boxed{2\cos(2x)}$

The graph of f, f', and f'' are shown below. Label each graph correctly and explain your reasoning.



+ slope
f

Key: when f max/min, $f' = 0$

- slope

(deriv not shown - must be f'')

Patterns

Given $f(x) = \frac{1}{x} = x^{-1}$, find a formula for the nth derivative $f^{(n)}(x)$.

$f'(x) = -1x^{-2}$

$f''(x) = +(2\cdot1)x^{-3}$

$f'''(x) = -(3\cdot2\cdot1)x^{-4}$

$f^{(4)}(x) = +(4\cdot3\cdot2\cdot1)x^{-5}$

$f^{(5)}(x) = -(5\cdot4\cdot3\cdot2\cdot1)x^{-6}$

$\vdots$

$$f^{(n)}(x) = (-1)^n n!\, x^{-(n+1)}$$

n factorial

$n! = n(n-1)(n-2)\cdots(3)(2)(1)$

Alternate Signs

$(-1)^n$ since even +

$(-1)^{n+1}$ if odd +)

(BONUS Example): Given $xy^3 + x^3y = 1$, find y''

$$y^3 + 3xy^2y' + 3x^2y + x^3y' = 0$$

$$3xy^2y' + x^3y' = -y^3 - 3x^2y$$

$$y' = \frac{-y^3 - 3x^2y}{3xy^2 + x^3}$$

$$y'' = \frac{(3xy^2 + x^3)(-3y^2y' - 6xy - 3x^2y') - (-y^3 - 3x^2y)(3y^2y' + 6xy + 3x^2y')}{(3xy^2 + x^3)^2}$$

$$= (3xy^2 + x^3)\left(-3y^2 \cdot \frac{-y^3 - 3x^2y}{3xy^2 + x^3} - 6xy - 3x^2 \cdot \frac{-y^3 - 3x^2y}{3xy^2 + x^3}\right) - \cdots$$

Trig Values

First Quadrant:

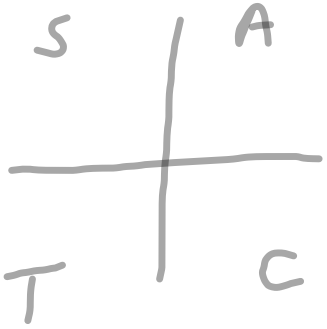
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos x$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
$\tan x$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞ (undef)

etc

Outside QI

① reference angle (∠ with x-axis)

② sign (+/−)



(and reciprocals)

Ex $\sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right)$

$= -\frac{1}{2}$