$\vec{r}\,'(t) = x'(t)\vec{i} + y'(t)\vec{j}$

3.9-Slopes and Tangents of Parametrized Curves

To find the slope of the tangent line for a parametrized curve, use the fact that

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad *$$

Examples: need slope, point

Find an equation of the line tangent to the curve given by $x = \sec\theta$, $y = \tan\theta$ at the point where $\theta = \dfrac{\pi}{3}$

point: $\theta = \frac{\pi}{3}$, $x = \sec\frac{\pi}{3} = 2$ $\quad y = \tan\frac{\pi}{3} = \sqrt{3}$ $\quad (2, \sqrt{3})$

slope: $$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sec^2\theta \;\frac{1}{c^2}}{\frac{1}{c}\sec\theta\tan\theta\,\frac{s}{c}} = \frac{1}{\sin\theta}$$

$$m = \frac{1}{\sin\frac{\pi}{3}} = \frac{2}{\sqrt{3}}$$

Equation: $\boxed{y - \sqrt{3} = \frac{2}{\sqrt{3}}(x-2)}$

Find an equation of the line tangent to the curve given by $x = \sqrt{t}$, $y = 2t + 4$ at the point $(3, 22)$.

Point: $(3, 22)$

Slope: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{\frac{1}{2}t^{-1/2}} = 4\sqrt{t}$

Find t: $\sqrt{t}^2 = 3^2$, $t = 9$ $\quad$ $2t+4=22$, $t=9$

$m = 4\sqrt{9} = 12$

Equation: $\boxed{y - 22 = 12(x-3) \quad \text{OR} \quad y = 12x - 14}$

Find the points on the curve $x = 2t^3 - 15t^2 + 24t + 7$, $y = t^2 + t + 1$ where the tangent line is horizontal or vertical

$m = 0$ 'm' undef

$\frac{dy}{dx} = 0$

Horiz

$\frac{dy/dt}{dx/dt} = 0$ when $\frac{dy}{dt} = 0$

$2t + 1 = 0$

$t = -\frac{1}{2}$

$x = -\frac{1}{4} - \frac{15}{4} - 12 + 7 = -9$

$y = \frac{1}{4} - \frac{1}{2} + 1 = \frac{3}{4}$

$\boxed{H \quad \left(-9, \frac{3}{4}\right)}$

$\frac{dy}{dx}$ undef

Vert

$\frac{dy/dt}{dx/dt}$ undef when $\frac{dx}{dt} = 0$

$6t^2 - 30t + 24 = 0$

$6(t^2 - 5t + 4) = 0$

$6(t-1)(t-4) = 0$

$t = 1 \quad t = 4$

$x = 2 - 15 + 24 + 7 = 18$

$y = 3$

$x = 128 - 240 + 96 + 7$

$y = 16 + 4 + 1$

$\boxed{V \quad (18, 3) \quad (-9, 21)}$

Summary: Horiz Tangents when $\frac{dy}{dt} = 0$ $\left(\frac{dx}{dt} \neq 0\right)$

Vert Tangents when $\frac{dx}{dt} = 0$ $\left(\frac{dy}{dt} \neq 0\right)$

Point, Slope

The curve $x = t^3 - 4t$, $y = t^2$ crosses itself at the point $(0, 4)$. Find equations of both tangent lines.

Point $(0,4)$

Slope: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2-4}$

Find t: $t^3 - 4t = 0$ $\quad t^2 = 4$

$t(t^2-4) = 0$ $\quad t = \pm 2$

$t = 0,\ t = 2,\ t = -2$

± 2 solve both equations

$t=2$ $\quad m = \frac{4}{3(4)-4} = \frac{1}{2}$ $\quad$ Equation: $y = \frac{1}{2}x + 4$

$t=-2$ $\quad m = \frac{-4}{3(4)-4} = -\frac{1}{2}$ $\quad$ Equation $y = -\frac{1}{2}x + 4$