4.3-Logarithmic Functions

inverse of exponential function

Definition: A *logarithmic function* is defined as follows:

If $y = \log_a x$, then $x = a^{y}$ logarithm answers question "what exponent?"

$y = \ln x$ implies $x = e^y$

Properties of Logarithms:

$\log_a(XY) = \log_a X + \log_a Y$

NOTE
$\log_a(X \pm Y)$ has NO identity!

$\log_a\left(\frac{X}{Y}\right) = \log_a X - \log_a Y$

$\log_a(X^n) = n \log_a X$

$\log_a x = y$ means $a^y = x$

Rewrite $\log_4 16 = 2$ as an exponential equation.

$4^2 = 16$

Compute $\log_2 \frac{1}{32}$

Method I

$y = \log_2 \frac{1}{32}$

means $2^y = \frac{1}{32}$

$= \frac{1}{2^5} = 2^{-5}$

$\boxed{y = -5}$

Method II

$\log_2\left(\frac{1}{32}\right) = \log_2 1 - \log_2 32$

$= 0 - 5$

since $2^0 = 1$ since $2^5 = 32$

$= \boxed{-5}$

Compute $\log_9 27$

$\log_9 27 = y$ means

$9^y = 27$ convert to common base

$(3^2)^y = 3^3$

$3^{2y} = 3^3$

$2y = 3$

$\boxed{y = \frac{3}{2}}$

$9^{3/2} = (\sqrt{9})^3 = 3^3 = 27$

$\log_a x = y$ means $a^y = x$

Given $\log_5 x = -3$, find x

$5^{-3} = x$

$x = \frac{1}{5^3}$

$\boxed{x = \frac{1}{125}}$

Rewrite $\ln X - 2 \ln Y + \frac{1}{2} \ln Z$ as a single logarithm

$= \ln X - \ln Y^2 + \ln Z^{1/2}$

$= \ln\left(\frac{X}{Y^2}\right) + \ln Z^{1/2}$

$= \boxed{\ln\left(\frac{X Z^{1/2}}{Y^2}\right)}$

Compute $5\log_{10} 2 + 2\log_{10} 5 - \log_{10} 8$

$= \log 2^5 + \log 5^2 - \log 8$

$= \log(2^5 \cdot 5^2) - \log 8$

$= \log\left(\frac{\cancel{32}\,4 \cdot 25}{\cancel{8}}\right)$

$= \log_{10} 100$

$= \boxed{2}$ since $10^2 = 100$

Find the inverse of $f(x) = e^{\frac{1}{x}}$

$\ln x = \ln e^{\frac{1}{y}}$

$\ln x = \frac{1}{y}$

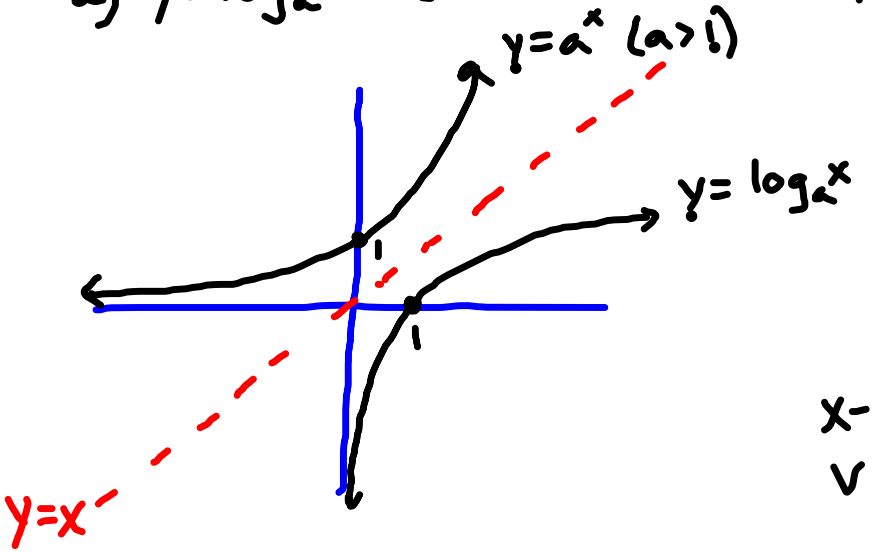
$$\boxed{f^{-1}(x) = y = \frac{1}{\ln x}}$$

Alt: $\ln(e^{\frac{1}{y}}) = \frac{1}{y}\ln e = \frac{1}{y}(1)$ since $e^1 = e$

Graphs of Logarithmic Functions:

1) The graph of $y=f^{-1}(x)$ is a reflection of $y=f(x)$ about the line $y=x$.

2) $y=\log_a x$ is the inverse of $y=a^x$



D : $x\in(0,\infty)$

R: $y\in(-\infty,\infty)$

x-Int $(1,0)$

V Asym $x=0$ $\quad \lim\limits_{x\to 0^+} \log_a x = -\infty$

$\lim\limits_{x\to\infty} \log_a x = \infty$

$\left.\begin{array}{l}\text{cts}\\ \text{Diff}\end{array}\right\}$ on domain

Examples:

Solve for x: $\log(2-x)+\log(5-x)=1$

$$\log[(2-x)(5-x)]=1$$

means $$(2-x)(5-x)=10^1$$

$$10-7x+x^2=10$$

$$x^2-7x=0$$

$$x(x-7)=0$$

$$\boxed{x=0} \quad \cancel{x=7}$$

Domain Restriction

$$\log(\cancel{2-7})+\log(\cancel{5-7})$$

$$\log(2-0)+\log(5-0)$$

$$\lim_{x\to-\infty} \ln(e^x + e^{-x}) - \ln(2e^x + e^{-x})$$

$$= \lim_{x\to-\infty} \ln\left(\frac{e^x+e^{-x}}{2e^x+e^{-x}}\right)$$

Since ln x is cts

$$= \ln\left(\lim_{x\to-\infty} \frac{e^x+e^{-x}}{2e^x+e^{-x}}\right)$$

Divide by dominating term e^{-x}

$$= \ln\left(\lim_{x\to-\infty} \frac{\cancel{e^{-x}}(\underbrace{e^{2x}}_{\to 0}+1)}{\cancel{e^{-x}}(\underbrace{2e^{2x}}_{\to 0}+1)}\right)$$

$$= \ln 1$$

$$= \boxed{0}$$

NOTE: You can show

$$\lim_{x\to\infty} \ln(e^x+e^{-x}) - \ln(2e^x+e^{-x}) = -\ln 2$$

The formula to compute the amount of money A in an account earning $100r\%$ interest compounded m times per year after t years is $A = P\left(1+\frac{r}{m}\right)^{mt}$. If 10,000 QR are kept at 6% per year compounded monthly, when will the account have 15,000 QR?

P

$r = .06$

$m = 12$

A

$$\frac{15000}{10000} = \frac{10000}{10000}\left(1+\frac{.06}{12}\right)^{12t}$$ solve for t

$$\log_{1.005} 1.5 = \log_{1.005}(1.005)^{12t}$$

$$\log_{1.005} 1.5 = 12t$$

$$t = \frac{1}{12}\log_{1.005} 1.5$$

$$\ln 1.5 = \ln(1.005)^{12t}$$

$$\ln 1.5 = 12t \ln 1.005$$

$$\frac{\ln 1.5}{\ln 1.005} = 12t$$

$$t = \frac{1}{12}\frac{\ln 1.5}{\ln 1.005} \approx$$

$$\log_{1.005} 1.5 = \frac{\ln 1.5}{\ln 1.005}$$

The *Change of Base* formula: $\boxed{\log_a x = \frac{\ln x}{\ln a}}$