4.4-Derivatives of Logarithmic Functions

Why do we know that the function $f(x) = \ln x$ is differentiable?

$g(x) = \ln x$ is the inverse of the differentiable function $f(x) = e^x$.

$y = \ln x$ means

$x = e^y$

$1 = e^y \cdot y'$

$y' = \frac{1}{e^y} = \frac{1}{x}$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

Other Bases: $y = \log_a x = \dfrac{\ln x}{\ln a}$

$$y' = \frac{1}{\ln a} \cdot \frac{1}{x}$$

$$\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}}$$

Logarithmic Differentiation:

1. Use to differentiate $f(x)^{g(x)}$ (REQUIRED) or lots of products, quotients, and powers (OPTIONAL)
2. Take ln of both sides of $y = f(x)$
3. Differentiate implicitly

Examples:

Compute and simplify:

$$\frac{d}{dx}(\ln(-x)) \overset{\text{Chain Rule}}{=} \frac{1}{-x}\cdot(-1)$$

$$= \frac{1}{x}$$

NOTE if $x>0$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

if $x<0$ $\frac{d}{dx}(\ln(-x)) = \frac{1}{x}$

$\therefore \frac{d}{dx}(\ln|x|) = \frac{1}{x}$

$$\frac{d}{dx}(\ln|\sin x|) = \frac{1}{\sin x}\cdot\cos x = \frac{\cos x}{\sin x} = \cot x$$

In 4.1 we proved that, if $f(x) = a^x$, then $f'(x) = Ka^x$, where $K = f'(0)$. Use logarithmic differentiation to find K.

$$\ln y = \ln(a^x)$$

$$\ln y = x \cdot \ln a$$

$$\frac{y'}{y} = \ln a$$

$$y' = y(\ln a)$$

$$y' = (\ln a)a^x$$

$$\frac{d}{dx}(a^x) = a^x \cdot \ln a$$

Given $f(x) = x^3 \ln|3 - 2x|$, find $f'(x)$ Product Rule

$$f'(x) = 3x^2 \cdot \ln|3-2x| + \frac{-2}{3-2x} \cdot x^3$$

Find the derivative of $f(x) = (1+x)^{\frac{1}{x}}$ Log Diff

$$\ln y = \ln(1+x)^{\frac{1}{x}}$$

$$\ln y = \frac{1}{x}\ln(1+x) = \frac{\ln(1+x)}{x}$$

$$\frac{y'}{y} = \frac{x\left(\frac{1}{1+x}\right) - \ln(1+x)(1)}{x^2}$$

$$y' = y\left[\frac{\frac{x}{1+x} - \ln(1+x)}{x^2}\right]$$

$$y' = (1+x)^{\frac{1}{x}}\left[\frac{\frac{x}{1+x} - \ln(1+x)}{x^2}\right]$$

Find the derivative of $f(x) = \dfrac{x^2 \sin x}{\sqrt{1+2x}}$

Quot, Prod, Power, Trig, Chain
OR Log Diff

$$\ln y = \ln\left(\frac{x^2 \sin x}{(1+2x)^{1/2}}\right)$$

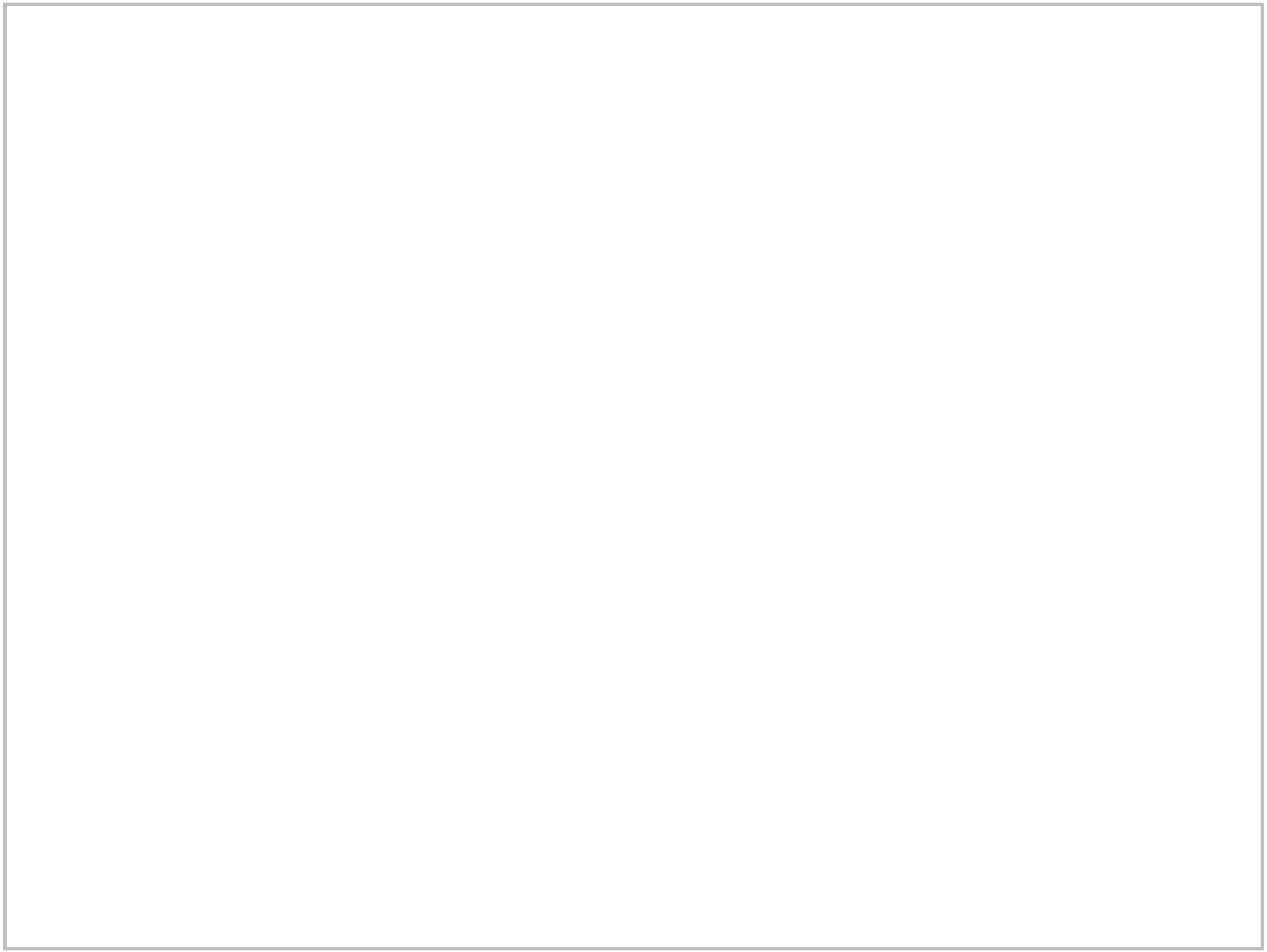
$$\ln y = \ln(x^2 \sin x) - \ln((1+2x)^{1/2})$$

$$\ln y = \ln(x^2) + \ln(\sin x) - \ln((1+2x)^{1/2})$$

$$\ln y = 2\ln x + \ln(\sin x) - \frac{1}{2}\ln(1+2x)$$

$$\frac{y'}{y} = \frac{2}{x} + \frac{\cos x}{\sin x} - \frac{1}{2}\cdot\frac{2}{1+2x}$$

$$y' = y\left[\frac{2}{x} + \frac{\cos x}{\sin x} - \frac{1}{1+2x}\right]$$



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