4.8 L’Hospital’s Rule

Goal: Given a limit of indeterminate form (0/0, ∞/∞, etc) with differentiable functions, find the limit.

L’Hospital’s Rule: If \( f \) and \( g \) are differentiable and \( g'(x) \neq 0 \) on an open interval \( I \) that contains \( a \) (except possibly at \( a \)), and \( \lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0 \) or \( \lim_{x \to a} f(x) = \pm \infty \) and \( \lim_{x \to a} g(x) = \pm \infty \), then

\[
\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}
\]

Examples:

Find the exact values of each of the following limits:

\[
\lim_{x \to -1} \frac{x^2 + 6x + 5}{x^2 - 3x - 4}
\]

\[
\lim_{x \to 0} \frac{1 - \cos x}{2x^2}
\]

\[
\lim_{x \to 0} x \ln x
\]
Recall the formula for computing compound interest (4.3): $A = P \left( 1 + \frac{r}{m} \right)^{mt}$. Find $\lim_{m \to \infty} A$.

On Your Own: 4.8 #3,9,11,13,17,19,21,22,26,30,37,39,44,47,54,56,57,61,62,65,77,78