

5.2-Maxima and Minima

Definitions:

f has a *relative maximum* at $x = a$ if and only if

f has a *relative minimum* at $x = a$ if and only if

Fermat's Theorem: If f has a relative maximum or relative minimum at $x = a$ and f is differentiable at $x = a$, then

More definitions:

f has a critical value at $x = a$ if and only if

f has an *absolute maximum* at $x = a$ if and only if

f has an *absolute minimum* at $x = a$ if and only if

Extreme Value Theorem If f is continuous on a closed, bounded interval, then

Graphical examples to show that each of the conditions must hold to guarantee the conclusion:

Examples:

Find the absolute maximum and absolute minimum of $f(x) = 2x^3 - 15x^2 + 36x + 7$ on the interval $[1, 5]$

Find the absolute maximum and absolute minimum of $f(x) = 2\sec x - \tan x$ on the interval $\left[0, \frac{\pi}{4}\right]$.

Find the absolute maximum and absolute minimum of $f(x) = \frac{\ln x}{x}$ on the interval $(0, \infty)$

Find the dimensions of a rectangle with perimeter 100 m whose area is as large as possible.