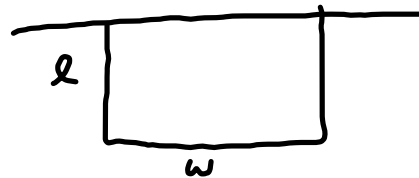


5.5-Applied Max/Min Problems

Goal: To optimize a practical value subject to certain restrictions (often given geometrically)

Examples:

A farmer has 1200 feet of fencing to enclose a field bordered by a river on one side. If no fence is needed along the river, find the dimensions of the field which give the largest area.



rectangular

Goal: $\text{Max } A = lw$

Restriction: $P = 1200 = 2l + w$
 solve for one variable

$w = 1200 - 2l$

$w \geq 0$

$A = l(1200 - 2l)$

$A = 1200l - 2l^2; 0 \leq l \leq 600$

$A' = 1200 - 4l = 0$

$l = 300$ so $w = 1200 - 2(300) = 600$

300 x 600 ft

***MUST SHOW MAX**

Three choices:

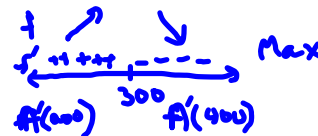
① Extreme Value Thm

$A(0) = 0$

$A(300) = 180,000$ Max

$A(600) = 0$

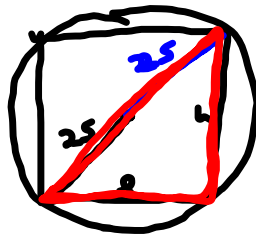
② First Der Test



③ Second Der Test

$A''(l) = -4$ \curvearrowright Max

A rectangular beam is to be cut from a cylindrical log of radius 25 cm. Find the dimensions which maximize the area of the rectangular cross-section of the beam.



Goal: Max $A = lw$ $a = A^2 = l^2 w^2$

Restriction: $l^2 + w^2 = 50^2$
 (right triangle)

$$w^2 = 2500 - l^2$$

$$w = \sqrt{2500 - l^2}$$

$$\text{max } A = l \sqrt{2500 - l^2}$$

$$a = l^2(2500 - l^2) = 2500l^2 - l^4 ; (0 \leq l \leq 50)$$

$$a' = 5000l - 4l^3 = 0$$

$$4l(1250 - l^2) = 0$$

$$l = 0 \quad l = \sqrt[3]{1250}$$

optional

Show max:

Second Der Test

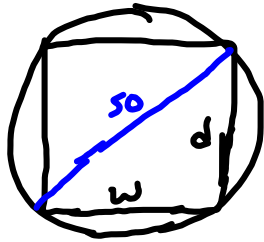
$$a'' = 5000 - 12l^2$$

$$5000 - 12(1250) < 0 \quad \text{Max}$$

$$l = \sqrt{1250} \quad w = \sqrt{2500 - 1250} = \sqrt{1250}$$

$$\boxed{\sqrt{1250} \times \sqrt{1250} \text{ cm}}$$

Suppose the strength of the beam in the previous example is proportional to the product of its width and the square of its depth. Find the dimensions of the strongest beam that can be cut from the log.



$$\text{Goal: Max } S = w d^2$$

$$\text{Restriction } d^2 + w^2 = 2500$$

$$d^2 = 2500 - w^2$$

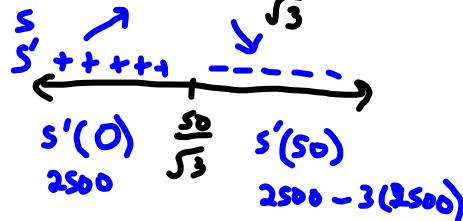
$$S = w(2500 - w^2) \\ = 2500w - w^3$$

$$S' = 2500 - 3w^2 = 0$$

$$w^2 = \frac{2500}{3}$$

$$w = \frac{50}{\sqrt{3}}$$

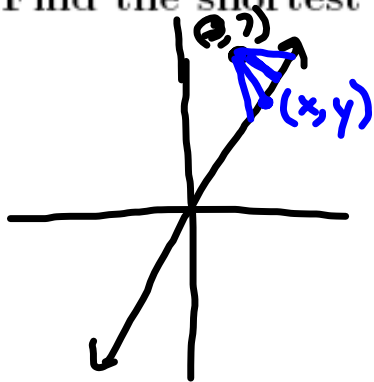
Show max
First Der Test:



$$w = \frac{50}{\sqrt{3}}, \text{ so } d = \sqrt{2500 - \frac{2500}{3}} = \sqrt{\frac{5000}{3}}$$

$$\boxed{\frac{50}{\sqrt{3}} \times \sqrt{\frac{5000}{3}} \text{ c.m}}$$

Find the shortest distance from the point $(3, 7)$ to the line $y = 2x$.



Goal $\min d = \sqrt{(x-3)^2 + (y-7)^2}$

Restriction: $y = 2x$

$$d = \sqrt{(x-3)^2 + (2x-7)^2}$$

$$D = d^2 = (x-3)^2 + (2x-7)^2 \quad \text{No restriction on } x$$

$$D' = 2(x-3)(1) + 2(2x-7)(2)$$

$$2x - 6 + 8x - 28 = 0$$

$$10x = 34$$

$$x = \frac{34}{10} = \frac{17}{5}$$

Show min:

Second Der Test

$$D'' = 10 \quad \cup \text{ min}$$

$$d = \sqrt{\left(\frac{17}{5} - 3\right)^2 + \left(\frac{34}{5} - 7\right)^2} = \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{1}{5}\right)^2} = \sqrt{\frac{5}{25}} = \boxed{\frac{1}{\sqrt{5}}}$$

If a projectile is fired from the ground at an angle θ with initial speed v_0 , the position of the projectile is given by $r(t) = \langle (v_0 \cos \theta)t, -\frac{1}{2}gt^2 + (v_0 \sin \theta)t \rangle$. Find the angle which maximizes the horizontal range of the projectile.



Goal: Max $x = (v_0 \cos \theta)t$

Restriction: $-\frac{1}{2}gt^2 + (v_0 \sin \theta)t = 0$ v_0, g constants!
Solve for t

$$t(-\frac{1}{2}gt + v_0 \sin \theta) = 0$$

$$t=0 \quad t = \frac{2v_0 \sin \theta}{g}$$

$$\text{Max } x = (v_0 \cos \theta) \left(\frac{2v_0 \sin \theta}{g} \right)$$

$$= \frac{2v_0^2}{g} \sin \theta \cos \theta$$

NOTE:
 $\frac{v_0^2}{g} (2 \sin \theta \cos \theta)$
 $\frac{v_0^2}{g} (\sin 2\theta)$ Max
 when $\sin 2\theta = 1$
 $2\theta = \frac{\pi}{2}$
 $\theta = \frac{\pi}{4}$

$$x' = \frac{2v_0^2}{g} (\cos \theta \cdot \cos \theta + \sin \theta (-\sin \theta)) = 0$$

$$\frac{2v_0^2}{g} (\cos^2 \theta - \sin^2 \theta) = 0$$

$$\cos^2 \theta = \sin^2 \theta$$

$$1 = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$$

$$\tan \theta = \pm 1$$

$$\theta = \frac{\pi}{4} \quad \text{since } 0 \leq \theta \leq \frac{\pi}{2}$$

Show max:

$$x(0) = 0$$

$$x\left(\frac{\pi}{4}\right) = \frac{2v_0^2}{g} \frac{\sqrt{2}}{2} \frac{\sqrt{2}}{2} = \frac{v_0^2}{g} \text{ Max}$$

$$x\left(\frac{\pi}{2}\right) = 0$$

Max angle = $\frac{\pi}{4}$ or 45°