5.7-Antiderivatives

F is an antiderivative of f if and only if $F'(x) = f(x)$

Antiderivative Rules:

$\int f(x)\,dx$ = antiderivative of f

Derivative	Original Function	Derivative	Original Function
$x^n; n \neq -1$	$\frac{1}{n+1}x^{n+1} + C$	$\csc x \cot x$	$-\csc x + C$
$f'(x) \pm g'(x)$	$f(x) \pm g(x) + C$	$x'(t)\vec{i} + y'(t)\vec{j}$	$x(t)\vec{i} + y(t)\vec{j} + \vec{C}$
$cf'(x)$	$cf(x) + C$	e^x (a^x)	$e^x + C$ $\left(\frac{1}{\ln a}a^x + C\right)$
$\sin x$	$-\cos x + C$	$\frac{1}{x}$	$\ln\lvert x\rvert + C$
$\cos x$	$\sin x + C$	$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\sec^2 x$	$\tan x + C$	$\frac{1}{1+x^2}$	$\tan^{-1} x + C$
$\csc^2 x$	$-\cot x + C$	$\frac{1}{x\sqrt{x^2-1}}$	$\sec^{-1} x + C$
$\sec x \tan x$	$\sec x + C$	m	$mx + C$

Examples:

Find the most general antiderivative of $f(x) = x - \sqrt[4]{x}$

$= x^1 - x^{\frac{1}{4}}$

$$\boxed{F(x) = \frac{1}{2}x^2 - \frac{4}{5}x^{\frac{5}{4}} + C}$$

$(1+x)\,x^{-1/2}$

Alt: $\frac{1}{\sqrt{x}} + \frac{x}{\sqrt{x}}$

Find $f(x)$ given $f'(x) = \dfrac{1+x}{\sqrt{x}}$ and $f(1) = 0$

initial condition to find C

$x=1,\ f(x)=0$

$f'(x) = x^{-1/2} + x^{\frac{1}{2}}$

$f(x) = 2x^{\frac{1}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$

$0 = 2(1)^{1/2} + \frac{2}{3}(1)^{3/2} + C$

$0 = 2 + \frac{2}{3} + C \longrightarrow C = -\frac{8}{3}$

$$\boxed{f(x) = 2x^{\frac{1}{2}} + \frac{2}{3}x^{3/2} - \frac{8}{3}}$$

Find $f(x)$ given $f''(x) = 1 + 2\sin x - \cos x$, $f(0) = 3$, and $f'(0) = 1$

$x=0 \quad f'(x)=1$

$f'(x) = x - 2\cos x - \sin x + C_1$

$1 = 0 - 2\cos 0 - \sin 0 + C_1$

$C_1 = 3$ NOTE: Constant $\neq$ Initial Value!

$f'(x) = x - 2\cos x - \sin x + 3$

$x=0 \quad f(x)=3$

$f(x) = \frac{1}{2}x^2 - 2\sin x + \cos x + 3x + C_2$

$3 = \frac{1}{2}(0)^2 - 2\sin 0 + \cos 0 + 3(0) + C_2$

$C_2 = 2$

$$\boxed{f(x) = \frac{1}{2}x^2 - 2\sin x + \cos x + 3x + 2}$$

x^{-1} Power Rule Does NOT work!

Find $f(x)$ given $f'(x) = e^x - \dfrac{1}{x}$ and $f(1) = 0$.

$x=1 \quad f(x)=0$

$f(x) = e^x - \ln|x| + C$

$0 = e^1 - \ln|1| + C$

$C = -e$

$$\boxed{f(x) = e^x - \ln|x| - e}$$

$$\vec{r} \xrightarrow{\text{der}} \vec{v} \xrightarrow{\text{der}} \vec{a}$$

$$\xleftarrow[\text{antider}]{} \quad \xleftarrow[\text{antider}]{}$$

The acceleration of a particle is given by $\mathrm{a}(t) = 2t\mathrm{i} + 3\mathrm{j}$. If the initial velocity is $\mathrm{i} - \mathrm{j}$ and the initial position is $< 1, 2 >$, find the position function $\mathrm{r}(t)$ of the particle.

$t = 0$

$\vec{v}(t) = \vec{i} - \vec{j}$

$$\vec{v}(t) = t^2\vec{i} + 3t\vec{j} + \vec{C}$$

$$\vec{i} - \vec{j} = 0^2\vec{i} + 3(0)\vec{j} + \vec{C} \qquad \vec{C} = \vec{i} - \vec{j}$$

$$\vec{v}(t) = t^2\vec{i} + 3t\vec{j} + 1\vec{i} - \vec{j} = (t^2+1)\vec{i} + (3t-1)\vec{j}$$

$$\vec{r}(t) = \left(\tfrac{1}{3}t^3 + t + C_x\right)\vec{i} + \left(\tfrac{3}{2}t^2 - t + C_y\right)\vec{j}$$

$$1\vec{i} + 2\vec{j} = \left(\tfrac{1}{3}(0)^3 + 0 + C_x\right)\vec{i} + \left(\tfrac{3}{2}(0)^2 - 0 + C_y\right)\vec{j}$$

$C_x = 1$

$C_y = 2$

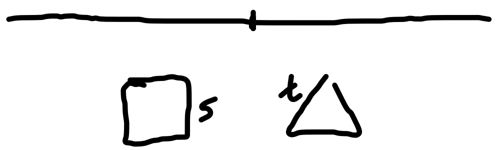
$$\boxed{\vec{r}(t) = \left(\tfrac{1}{3}t^3 + t + 1\right)\vec{i} + \left(\tfrac{3}{2}t^2 - t + 2\right)\vec{j}}$$

Group Lecture Quiz 5.5

Answer the following in groups of no more than 4 people. No notes, but CALCULATORS ARE ALLOWED!

A piece of wire 10m long is cut into at most two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be divided so that the total area enclosed by both figures is a maximum?

(NOTE: The area of an equilateral triangle is given by $A = \frac{t^2\sqrt{3}}{4}$, where t is the side length of the triangle).



Goal: Max $A = s^2 + \frac{t^2\sqrt{3}}{4}$

Restriction $4s + 3t = 10$

$s = \frac{1}{4}(10-3t)$

$A = \frac{1}{16}(10-3t)^2 + \frac{t^2\sqrt{3}}{4}$; $0 \le t \le \frac{10}{3}$

$A' = \frac{1}{8}(10-3t)(-3) + \frac{t\sqrt{3}}{2} = 0$

$\frac{9}{8}t - \frac{15}{4} + \frac{\sqrt{3}}{2}t = 0$

$t = \frac{15/4}{\frac{9}{8} + \frac{\sqrt{3}}{2}} \approx 1.88$

Extreme Val Thm (on Calc)

$A(0) = 6.25$

$A(1.88) = 2.72$

$A\left(\frac{10}{3}\right) = 4.81$

NOTE that crit value is a rel MINIMUM!

$\boxed{\text{Answer: entire wire goes to square}}$