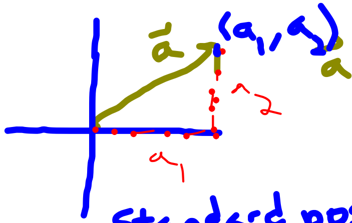
1.1-Vectors

Definitions:

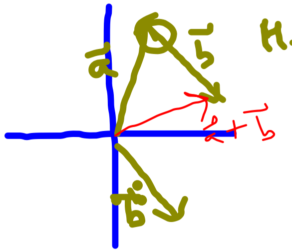
vector: A quantity with magnitude and direction



$\vec{a} = \langle a_1, a_2 \rangle$

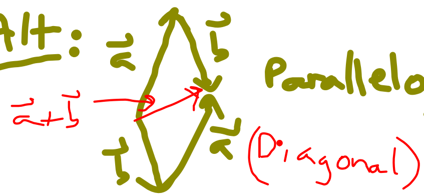
One to one correspondence between set of points and (2-D) vectors in standard position.

addition If $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$, then $\vec{a} + \vec{b} = \langle a_1 + b_1, a_2 + b_2 \rangle$

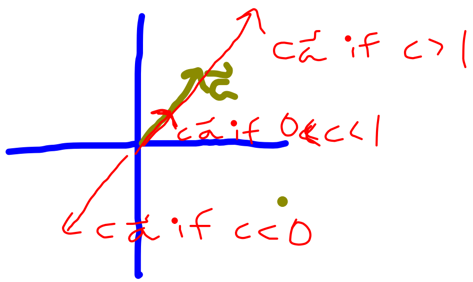


Head to Tail

Alt: Parallelogram

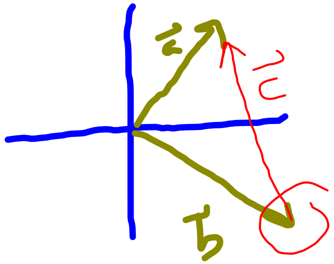


scalar multiplication If $\vec{a} = \langle a_1, a_2 \rangle$, then $c\vec{a} = \langle ca_1, ca_2 \rangle$



Parallel vectors → vectors are scalar multiples of each other.

[redacted] subtraction If $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$ then $\vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2 \rangle$

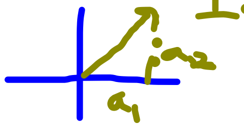


So $\vec{b} + \vec{c} = \vec{a}$

$\vec{c} = \vec{a} - \vec{b}$ "Head to Head"

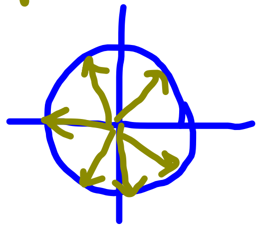
End − Start

magnitude: If $\vec{a} = \langle a_1, a_2 \rangle$, then $|\vec{a}| = \sqrt{a_1^2 + a_2^2}$



unit vector: A vector whose magnitude is 1

Every point on unit circle corresponds to a unit vector



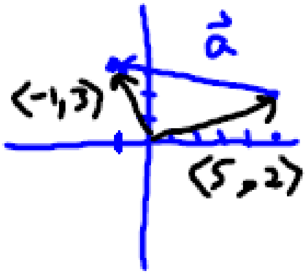
i and j: $\vec{i} = \langle 1, 0 \rangle$

$\vec{j} = \langle 0, 1 \rangle$

$$\begin{aligned}
\vec{a} &= \langle a_1, a_2 \rangle \\
&= \langle a_1, 0 \rangle + \langle 0, a_2 \rangle \\
&= a_1 \langle 1, 0 \rangle + a_2 \langle 0, 1 \rangle \\
&= \boxed{a_1 \vec{i} + a_2 \vec{j}}
\end{aligned}$$

Examples:

Find the components of the vector which begins at the point $(5, 2)$ and ends at the point $(-1, 3)$. Then find a unit vector parallel to this vector.



$$\vec{a} = \underbrace{\langle -1, 3\rangle}_{\text{End}} - \underbrace{\langle 5, 2\rangle}_{\text{Start}}$$

$$= \boxed{\langle -6, 1\rangle}$$

$$|\vec{a}| = \sqrt{(-6)^2 + 1^2}$$

$$= \sqrt{37}$$

$$\vec{a}_u = \frac{1}{\sqrt{37}}\langle -6, 1\rangle$$

$$= \boxed{\left\langle \frac{-6}{\sqrt{37}}, \frac{1}{\sqrt{37}} \right\rangle}$$

$5\vec{i}+2\vec{j}$

Given $\mathbf{a} = <5, 2>$ and $\mathbf{b} = -2\mathbf{i} + 4\mathbf{j}$, find each of the following:
$\mathbf{a}+\mathbf{b}$, $\mathbf{a}-\mathbf{b}$. $2\mathbf{a}$. $3\mathbf{a}+4\mathbf{b}$.

$\vec{a}+\vec{b} = (5+-2)\vec{i} + (2+4)\vec{j} = \boxed{3\vec{i}+6\vec{j}}$

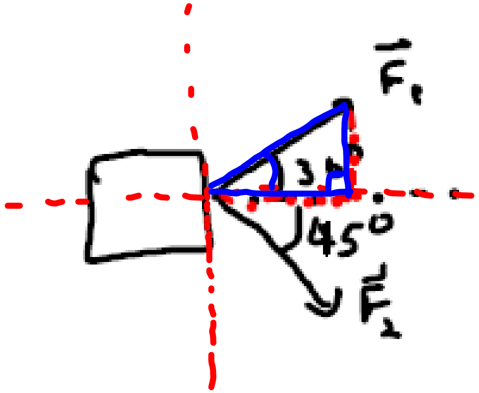
$\vec{a}-\vec{b} = (5-+2)\vec{i} + (2-4)\vec{j} = \boxed{7\vec{i}-2\vec{j}}$

$2\vec{a} = \boxed{10\vec{i}+4\vec{j}}$

$3\vec{a}+4\vec{b} = (15\vec{i}+6\vec{j}) + (-8\vec{i}+16\vec{j})$

$= \boxed{7\vec{i}+22\vec{j}}$

Two people are to pull ropes attached to a 50kg box (on a frictionless surface) as shown in the figure given in class. With what force does each person have to pull in order to have the box accelerate straight ahead at 0.5 meters per second squared?



resultant $\vec{F} = \vec{F}_1 + \vec{F}_2 = \overset{m}{(50)}\overset{a}{(0.5)}\,\vec{i} = 25\vec{i} + 0\vec{j}$

$|\vec{F}_1| = A \qquad |\vec{F}_2| = B$

$\vec{F}_1 = (A\cos\underset{\frac{\sqrt{3}}{2}}{30^\circ})\vec{i} + (A\sin\underset{\frac{1}{2}}{30^\circ})\vec{j}$

$\vec{F}_2 = (B\cos\underset{\frac{\sqrt{2}}{2}}{45^\circ})\vec{i} + (-B\sin\underset{\frac{\sqrt{2}}{2}}{45^\circ})\vec{j}$

$\vec{F} = \vec{F}_1 + \vec{F}_2 = 25\vec{i} + 0\vec{j}$: $\frac{\sqrt{3}}{2}A + \frac{\sqrt{2}}{2}B = 25$ ($\vec{i}$ component) solve for A, B

$+\ \frac{1}{2}A - \frac{\sqrt{2}}{2}B = 0$ ($\vec{j}$ component)

$2\cdot\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)A = 25\cdot 2$

$(\sqrt{3}+1)A = 50$

$\boxed{A = \frac{50}{\sqrt{3}+1}\ N}$ subs into 2nd equation

$\frac{1}{\cancel{2}}\cdot\frac{\overset{25}{\cancel{50}}}{\sqrt{3}+1} - \frac{\sqrt{2}}{2}B = 0$

$+\frac{\sqrt{2}}{2}B = +\frac{25}{\sqrt{3}+1}$

$\boxed{B = \frac{25}{\sqrt{3}+1}\cdot\frac{2}{\sqrt{2}}\ N}$