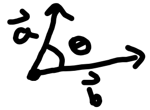
1.2-Dot Product



Definitions:

The *dot product* of the vectors a and b is given by $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Dot Product computation formula   if $\vec{a} = a_1\vec{i} + a_2\vec{j}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j}$, then

$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2$ NO $\vec{i}$ OR $\vec{j}$ (Scalar)

From the definition, it follows that the angle between two vectors is given by

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} \; ; \; \theta = \cos^{-1}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)$$

a and b are *orthogonal* if and only if $\vec{a}\cdot\vec{b} = 0$

$\perp$

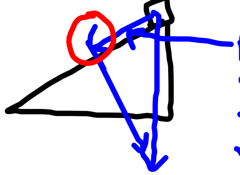
Orthogonal complements (only 2-D)

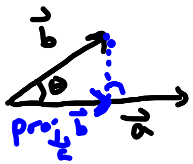
The orthogonal complement of $\vec{a} = a_1\vec{i} + a_2\vec{j}$ is

$$\vec{a}^{\perp} = -a_2\vec{i} + a_1\vec{j}$$

Scalar and Vector projections

Appl: Block on a Ramp





$$|\text{proj}_{\vec{a}}\vec{b}| = |\vec{b}|\cos\theta$$
$$= \cancel{|\vec{b}|}\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cancel{|\vec{b}|}}\right)$$

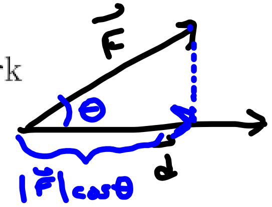
Scalar Projection

$$* \boxed{\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}} *$$

Vector Projection

$$* \boxed{\begin{aligned}\text{proj}_{\vec{a}}\vec{b} &= \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}\cdot\frac{1}{|\vec{a}|}\cdot\vec{a} \\ &= \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\,\vec{a}\end{aligned}} *$$

Work



$$W = |\vec{F}||\vec{d}|\cos\theta$$

$$W = \vec{F}\cdot\vec{d}$$

Examples:

$3\vec{i}+2\vec{j}$

Find $\mathbf{a}\cdot\mathbf{b}$ if $\mathbf{a} =< 3, 2 >$ and $\mathbf{b} = 1\mathbf{i} - 4\mathbf{j}$

$$\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2$$
$$= (3)(1) + (2)(-4)$$
$$= 3 - 8 = \boxed{-5}$$

Find the angle between $\mathbf{a} =< 3, 1 >$ and $\mathbf{b} =< -2, 4 >$.

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{(3)(-2)+(1)(4)}{\sqrt{3^2+1^2}\cdot\sqrt{(-2)^2+4^2}} = \frac{-2}{\sqrt{10}\cdot\sqrt{20}}$$
$$= \frac{-2}{\sqrt{200}} = \frac{-\cancel{2}^{\,1}}{\cancel{10}_{\,5}\sqrt{2}}$$

$$\theta = \boxed{\cos^{-1}\left(\frac{-1}{5\sqrt{2}}\right) \text{ or } \cos^{-1}\left(-\frac{\sqrt{2}}{10}\right)}$$

Find x such that $\mathbf{a} = < x, 1 > \perp \mathbf{b} = < 4 + x, 3 >$ Make $\vec{a} \cdot \vec{b} = 0$

$$\vec{a} \cdot \vec{b} = x(4+x) + (1)(3) = 0$$
$$4x + x^2 + 3 = 0$$
$$x^2 + 4x + 3 = 0$$
$$(x+3)(x+1) = 0$$ or use Quad Formula
$$\boxed{x = -3 \text{ or } x = -1}$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

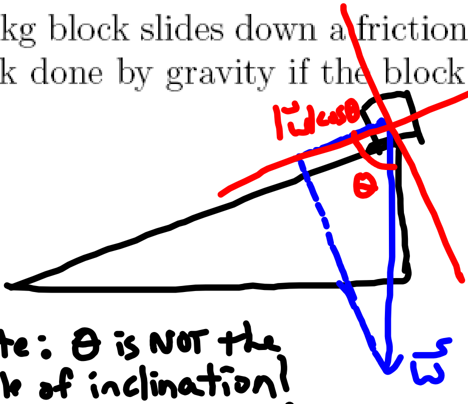
If $\mathbf{a} = \mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = < 4, 1 >$ ($4\vec{i} + \vec{j}$), find the scalar and vector projection of $\mathbf{a}$ onto $\mathbf{b}$

$$\text{comp}_{\vec{b}} \vec{a} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{(1)(4) + (3)(1)}{\sqrt{4^2 + 1^2}} = \boxed{\frac{7}{\sqrt{17}}}$$

$$\text{proj}_{\vec{b}} \vec{a} = \frac{7}{\sqrt{17}} \cdot \frac{1}{\sqrt{17}} \cdot (4\vec{i} + \vec{j})$$ or use $\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \vec{b}$

$$= \frac{7}{17}(4\vec{i} + \vec{j})$$
$$= \boxed{\frac{28}{17}\vec{i} + \frac{7}{17}\vec{j}}$$

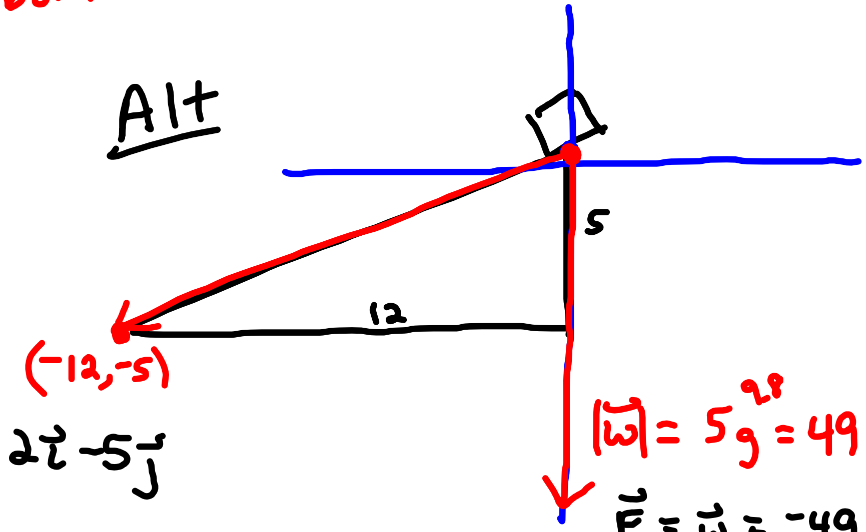
A 5 kg block slides down a frictionless ramp 5 meters tall and 12 meters long (horizontally). Find the work done by gravity if the block slides the entire length of the ramp.



Don't know θ

Note: θ is NOT the angle of inclination!

Alt



$$|\vec{w}| = 5\overset{9.8}{g} = 49\text{ N}$$

$$\vec{F} = \vec{w} = -49\,\vec{j}$$

$$\vec{d} = -12\vec{i} - 5\vec{j}$$

$$W = \vec{F}\cdot\vec{d}$$

$$= (0)(-12) + (-49)(-5)$$

$$= \boxed{245\text{ N}\cdot\text{m or J}}$$