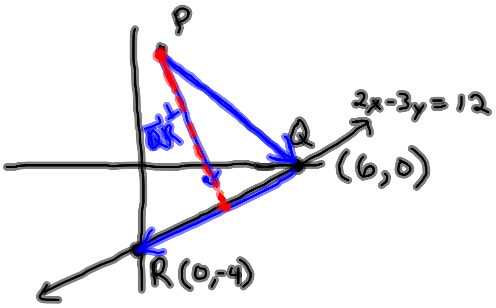
Bonus Example 1.2: Find the distance from the point (1, 5) to the line $2x - 3y = 12$.



Step 1: Find points Q and R on the line

Step 2:

$$d = \left| \operatorname{comp}_{\vec{QR}^{\perp}} \vec{PQ} \right|$$

$$\vec{PQ} = (6\vec{i} + 0\vec{j}) - (1\vec{i} + 5\vec{j}) = 5\vec{i} - 5\vec{j}$$

$$\vec{QR} = (0\vec{i} - 4\vec{j}) - (6\vec{i} + 0\vec{j}) = -6\vec{i} - 4\vec{j}$$

$$\vec{QR}^{\perp} = +4\vec{i} - 6\vec{j}$$

$$d = \left| \operatorname{comp}_{\vec{QR}^{\perp}} \vec{PQ} \right| = \left| \frac{\vec{QR}^{\perp} \cdot \vec{PQ}}{|\vec{QR}^{\perp}|} \right|$$

$$= \left| \frac{(4)(5) + (-6)(-5)}{\sqrt{4^2 + (-6)^2}} \right| = \boxed{\frac{50}{\sqrt{52}}}$$

1.3-Vector-Valued Functions and Parametrized Curves

Idea of a Vector Function/Parametrized Curve:

(Recall: a function is a rule that assigns to each input (domain) a unique output (range))

Vector function $\vec{r}: \mathbb{R} \to \mathbb{R}^2$ (2-dim vector) $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j}$

Graph is the set of all outputs (points corresponding to vectors)

$x = x(t)$ t = parameter ("when")

$y = y(t)$ (x,y) = position ("where") } Parametrized Curve

Eliminating the Parameter remove t from equations, leaving x, y (Cartesian)

1) if possible, solve one variable for t and substitute into other

2) if trig functions, use identities that relate the functions

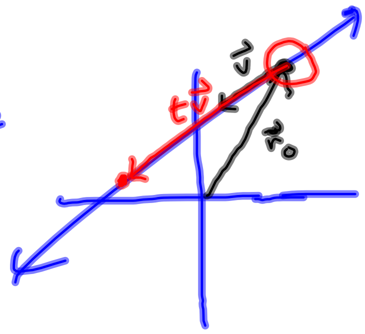
Vector and Parametric Equations of a Line Need point, slope

$\vec{r}_0$ = a vector corresponding to point

$\vec{v}$ = a vector in the direction of (parallel to) line

* $\boxed{\vec{r}(t) = \vec{r}_0 + t\vec{v}}$ * Vector Equation of a Line

$y = b + x m$



Examples:

Given $\mathbf{r}(t) = (t^{\frac{1}{2}} + 1)\mathbf{i} + t^{\frac{3}{2}}\mathbf{j}$:

a) Find $\mathbf{r}(1)$ and $\mathbf{r}(t+h)$

b) Eliminate the parameter and sketch the graph

c) When (if at all) does the graph pass through the point $(\overset{x}{3}, \overset{y}{8})$?

a) $\vec{r}(1) = (\overset{\sqrt{1}}{1^{1/2}} + 1)\vec{i} + (1^{3/2})\vec{j}$

$= \boxed{2\vec{i} + 1\vec{j}}$ (2,1) is on graph

$\vec{r}(t+h) = \boxed{((t+h)^{1/2} + 1)\vec{i} + (t+h)^{3/2}\vec{j}}$

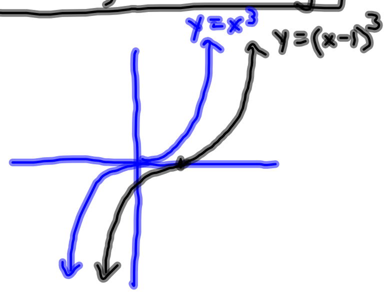
b) $x = t^{1/2} + 1 \longrightarrow (x-1)^2 = (t^{1/2})^2$

$y = t^{3/2}$

$t = (x-1)^2$

$y = ((x-1)^2)^{3/2}$

$y = (x-1)^3$



c) $3 = t^{1/2} + 1$

$2 = t^{1/2}$

$t = 4 \longrightarrow$

$8 = t^{3/2}$

$8 = 4^{3/2}$?

$8 = \sqrt{4^3}$?

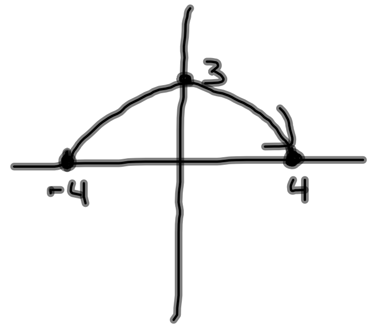
$8 = \sqrt{64}$ ✓

$\boxed{\text{Yes, it passes } (3,8) \text{ when } t=4}$

Describe the motion of a particle whose position is given by $x = -4\cos t,\ y = 3\sin t,\ 0 \le t \le \pi$

1) Start $t=0$

$x = -4\cos 0 = -4$
$y = 3\sin 0 = 0$



2) End $t=\pi$

$x = -4\cos\pi = 4$
$y = 3\sin\pi = 0$

3) Path (eliminate parameter)

$\sin^2 t + \cos^2 t = 1$

$y = 3\sin t$
$\frac{y}{3} = \sin t$

$\frac{y^2}{9} + \frac{x^2}{16} = 1$

$x = -4\cos t$
$\frac{x}{-4} = \cos t$

Ellipse

4) Direction, if necessary
pick a t in interval $t = \frac{\pi}{2}$

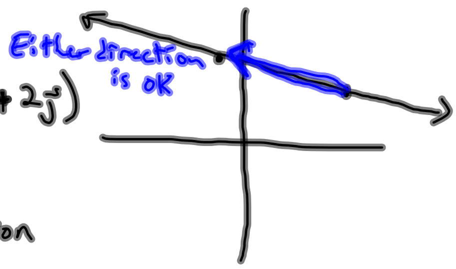
$x = -4\cos\frac{\pi}{2} = 0$
$y = 3\sin\frac{\pi}{2} = 3$

upper half of ellipse

Find vector and parametric equations of the line passing through the points $(-1, 3)$ and $(5, 2)$

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$

$\vec{r}_0 = -1\vec{i} + 3\vec{j}$ Either point is OK



$\vec{v} =$
$(-1\vec{i}+3\vec{j}) - (5\vec{i}+2\vec{j})$
$= -6\vec{i} + \vec{j}$

$\boxed{\vec{r}(t) = (-1\vec{i}+3\vec{j}) + t(-6\vec{i}+\vec{j})}$ Vector Equation

(Expand to find parametric)

$\vec{r}(t) = (-1\vec{i}+3\vec{j}) + (-6t\vec{i} + t\vec{j})$

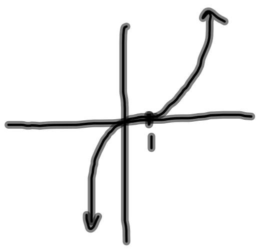
$= \boxed{(-1-6t)\vec{i} + (3+t)\vec{j}}$ OR $\boxed{\begin{aligned} x &= -1-6t \\ y &= 3+t \end{aligned}}$

Eliminate the parameter to sketch the graph of the vector function $\mathbf{r}(t) = \overset{x}{t}\mathbf{i} + \overset{y}{(t-1)^3}\mathbf{j}$. Does this graph differ from the first example? How?

$x = t \longrightarrow t = x$

$y = (t-1)^3$

$y = (x-1)^3$



Recall (1st example)

$x = \overset{\sqrt{t}+1}{\left(t^{1/2}+1\right)}$

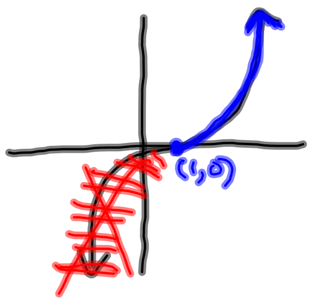
$y = t^{3/2} = \sqrt{t^3}$

Same equation $y = (x-1)^3$; $x \geq 1$

Domain: $t \geq 0$

Range of x: $x \geq 1$

Range of y: $y \geq 0$



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