2.3-Analytic Computation of Limits

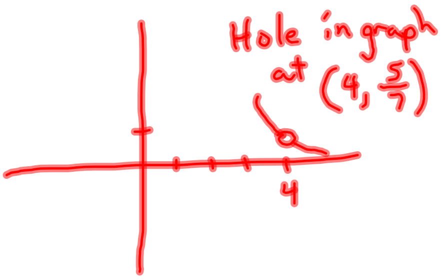
Properties of Limits: (pp 91-93. Basis for the techniques used in the following examples.)

Examples:

If polynomials, factor and cancel

$$\lim_{x\to 4} \frac{x^2-3x-4}{2x^2-9x+4} \quad \frac{16-12-4}{32-36+4} = \frac{0}{0}$$

$$= \lim_{x\to 4} \frac{(x+1)\cancel{(x-4)}}{\cancel{(x-4)}(2x-1)} = \frac{5}{7}$$



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$\lim_{t \to 1} (\sqrt{t})\mathbf{i} + (4t - 9)\mathbf{j}$

Key: $\lim_{t \to a} x(t)\vec{i} + y(t)\vec{j}$

$= \left(\lim_{t \to a} x(t)\right)\vec{i} + \left(\lim_{t \to a} y(t)\right)\vec{j}$

$= \left(\lim_{t \to 1} \sqrt{t}\right)\vec{i} + \left(\lim_{t \to 1} 4t-9\right)\vec{j}$

$= \boxed{1\vec{i} + {}^{-}5\vec{j}}$

OR

$\boxed{\vec{i} - 5\vec{j}}$

NOTE

$\sqrt{a}$ is always positive

$\sqrt{x^2} = \sqrt{4}$

$|x| = 2$ Usually skipped

$x = \pm 2$

If radicals, rationalize

$$\lim_{x\to 9}\frac{\sqrt{x}-3}{9-x} \quad \frac{0}{0}$$

$$= \lim_{x\to 9}\frac{(\sqrt{x}-3)(\sqrt{x}+3)}{(9-x)(\sqrt{x}+3)}$$

$$= \lim_{x\to 9}\frac{x-9}{(9-x)(\sqrt{x}+3)}$$

$$= \lim_{x\to 9}\frac{\cancel{x-9}}{-\cancel{(x-9)}(\sqrt{x}+3)} = \frac{1}{-(\sqrt{9}+3)} = \boxed{-\frac{1}{6}}$$

Hole in graph at $\left(9, -\frac{1}{6}\right)$

$$\lim_{h\to 0}\frac{\frac{2}{x+h}-\frac{2}{x}}{h} \quad \frac{0}{0}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{2\,x}{(x+h)x} - \frac{2(x+h)}{x(x+h)}\right) \quad 2x+2h$$

$$= \lim_{h\to 0} \frac{1}{h}\left(\frac{\cancel{2x}-\cancel{2x}-2h}{x(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{1}{\cancel{h}}\left(\frac{-2\cancel{h}}{x(x+h)}\right)$$

$$= \lim_{h\to 0} \frac{-2}{x(x+h)} = \frac{-2}{x^2}$$

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$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$$

Squeeze Theorem: If $g(x) \le f(x) \le h(x)$ and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$.

We know $-1 \le \sin\left(\frac{1}{x}\right) \le 1$ multiply everything by x^2 to get our given function in middle

$$-x^2 \le x^2\sin\left(\frac{1}{x}\right) \le x^2$$

$$\lim_{x\to 0} -x^2 = 0 \qquad \lim_{x\to 0} x^2 = 0$$

$\therefore$ by Squeeze Theorem, $\lim_{x\to 0} x^2\sin\left(\frac{1}{x}\right) = 0$.