2.5-Continuity

Definitions:

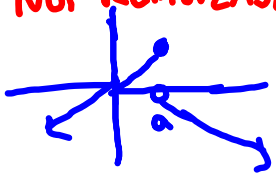
f is *continuous* at $x = a$ if and only if $\lim_{x \to a} f(x) = f(a)$

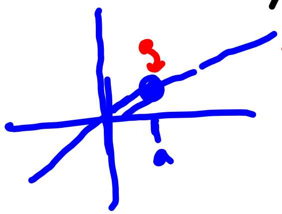
3 point checklist:

1) f defined at x=a (y-value)?
2) limit exist at x=a?
3) are they equal? (y-value = limit)

Removable Discontinuities: f has a removeable discontinuity at x=a if f is not cts at x=a, but there is a cts function g such that $g(x) = f(x)$ for all $x \neq a$.

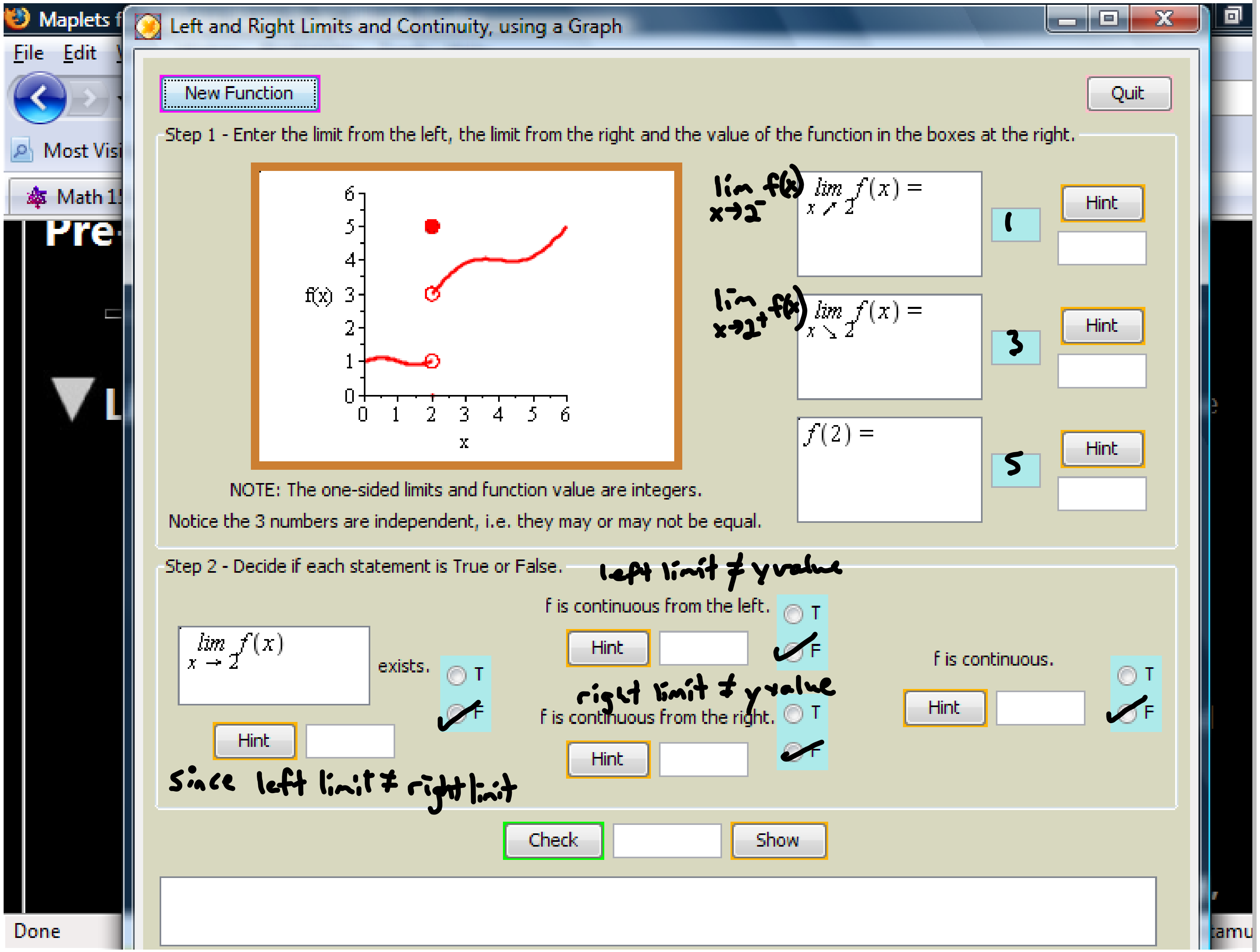
NOT REMOVEABLE





REMOVEABLE

(NOTE: removeable if the limit exists!)


Left and Right Limits and Continuity, using a Graph
New Function
Quit
Step 1 - Enter the limit from the left, the limit from the right and the value of the function in the boxes at the right.
f(x)
x
lim f(x) x→2⁻
$\lim_{x \nearrow 2} f(x) =$
1
Hint
lim f(x) x→2⁺
$\lim_{x \searrow 2} f(x) =$
3
Hint
$f(2) =$
5
Hint
NOTE: The one-sided limits and function value are integers.
Notice the 3 numbers are independent, i.e. they may or may not be equal.
Step 2 - Decide if each statement is True or False.
left limit ≠ y value
f is continuous from the left.
T
F
Hint
$\lim_{x \to 2} f(x)$ exists.
T
F
Hint
right limit ≠ y value
f is continuous from the right.
T
F
Hint
f is continuous.
T
F
Hint
Since left limit ≠ right limit
Check
Show
Done

Theorems:

Limits inside Continuous Functions If f is cts, then

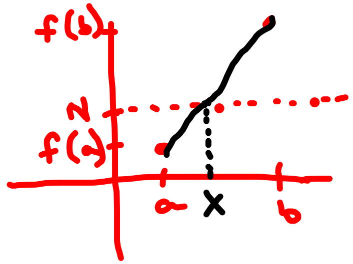
$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

Continuity of Polynomial/Rational Functions

cts on their domains

Intermediate Value Theorem

If f is cts on $[a,b]$ and N is between $f(a)$ and $f(b)$, then there is a solution to the equation $f(x)=N$ in the interval $[a,b]$



Examples:

If $f(x) = \begin{cases} 1-x & \text{if } x \geq 1 \\ -x & \text{if } x < 1 \end{cases}$

determine whether f is continuous at $x=1$ or not. Explain your answer precisely.

$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} -x = -1$

$(x<1)$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 1-x = 0$

$(x>1)$

$f(1) = 0$

Checklist:

1) y-value? YES $(y=0)$
2) limit? NO left $\neq$ right

$\boxed{\text{Not cts since limit DNE}}$

NOTE: f is cts from right

If $f(x) = \begin{cases} \dfrac{x^2-9}{x-3} & \text{if } x \neq 3 \\ 1 & \text{if } x = 3 \end{cases}$

don't need left and right since same equation for $x<3$ and $x>3$

determine whether f is continuous at $x=3$ or not. Explain your answer precisely.

Checklist

1) y-value? $f(3) = 1$ YES

2) limit? $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{x^2-9}{x-3}$ $\frac{0}{0}$

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = 6$ YES

3) equal? NO $1 \neq 6$

$\boxed{\text{So } f \text{ is not cts since limit} \neq \text{y-value}}$

NOTE: f has a removeable discontinuity

cos is cts, so

Compute $\lim_{x\to 2} \cos\left(\frac{x-2}{x^2-4}\pi\right)$

$$= \cos\left(\lim_{x\to 2} \frac{x-2}{x^2-4}\cdot\pi\right)$$

0

0

$$= \cos\left(\lim_{x\to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)}\cdot\pi\right)$$

$$= \cos\left(\frac{1}{4}\cdot\pi\right)$$

$$= \cos\left(\frac{\pi}{4}\right)$$

$$= \boxed{\frac{\sqrt{2}}{2}}$$

Prove the existence of the number $\sqrt{2}$ by proving there is at least one solution to the equation $x^2 = 2$.

IVT If f cts on [a, b] and N is between f(a) and f(b), there is a solution to $f(x) = N$ on [a, b]

Let $f(x) = x^2$ cts
and $N = 2$

Let $a = 1$ $f(1) = 1$
$b = 2$ $f(2) = 4$

2 is between 1 and 4

$\therefore$ there is a solution to $x^2 = 2$ on [1, 2] by IVT.