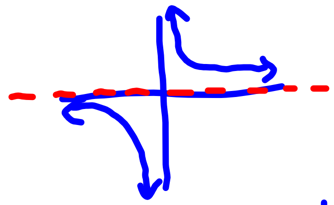
not 0 → $\frac{\#}{0}$

2.6-Limits at Infinity

In 2.2, we learned that if $y \to \pm\infty$ as $x \to a$, then the graph of f has a *vertical asymptote* at $x = a$. Similarly, if $y \to L$ as $x \to \pm\infty$, then the graph of the function has a *horizontal asymptote* at $y = L$.



Key Limit: $\lim_{x\to\infty} \frac{1}{x} = 0$ $\lim_{x\to-\infty} \frac{1}{x} = 0$

Computing limits at Infinity: (fraction)

Step 1) Factor out the dominating term ← highest exponent
from the numerator and denominator

Step 2) Cancel and apply key limit

Examples:

$$\frac{5\cancel{x}}{x^{\cancel{2}}}$$

$$\lim_{x\to\infty}\frac{3x^2-5x-1}{7+2x-4x^2} = \lim_{x\to\infty}\frac{\cancel{x^2}\left(3-\frac{5}{x}-\frac{1}{x^2}\right)}{\cancel{x^2}\left(\frac{7}{x^2}+\frac{2}{x}-4\right)}$$

$$= \lim_{x\to\infty}\frac{3-\underbrace{\cancel{\tfrac{5}{x}}}_{\to 0}-\underbrace{\cancel{\tfrac{1}{x^2}}}_{\to 0}}{\underbrace{\cancel{\tfrac{7}{x^2}}}_{\to 0}+\underbrace{\cancel{\tfrac{2}{x}}}_{\to 0}-4}$$

$$= \boxed{-\frac{3}{4}}$$

NOTE:

$$\lim_{x\to\infty}\frac{3x^2-5x-1}{7+2x-4x^3} = \lim_{x\to\infty}\frac{\cancel{x^2}\left(3-\underbrace{\cancel{\tfrac{5}{x}}}_{\to 0}-\underbrace{\cancel{\tfrac{1}{x^2}}}_{\to 0}\right)}{x^{\cancel{3}}\left(\underbrace{\cancel{\tfrac{7}{x^3}}}_{\to 0}+\underbrace{\cancel{\tfrac{2}{x^2}}}_{\to 0}-4\right)}$$

$$= \lim_{x\to\infty}\underbrace{\cancel{\frac{1}{x}}}_{\to 0}\cdot\frac{-3}{4} = \boxed{0}$$

Rationalize

Compute $\lim_{x\to\infty} \dfrac{\left(\sqrt{x^2+4x}-x\right)}{1} \dfrac{\left(\sqrt{x^2+4x}+x\right)}{\left(\sqrt{x^2+4x}+x\right)}$

$$= \lim_{x\to\infty} \frac{(\cancel{x^2}+4x)-\cancel{x^2}}{\sqrt{x^2+4x}+x}$$

$$= \lim_{x\to\infty} \frac{4x}{\sqrt{x^2+4x}+x}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}(4)}{\cancel{x}\left(\sqrt{1+\frac{4}{x}}+1\right)}$$

$$\frac{\sqrt{x^2+4x}}{x} = \sqrt{\frac{x^2+4x}{x^2}} = \sqrt{\frac{x^2}{x^2}+\frac{4x}{x^2}}$$

$$= \lim_{x\to\infty} \frac{4}{\sqrt{1+\frac{4}{x}}+1} = \frac{4}{1+1} = \boxed{2}$$

$$\frac{4}{x} \to 0$$

$\sqrt{x^2} = |x| = -x$

Find the horizontal asymptotes of $f(x) = \dfrac{\sqrt{2x^2+1}}{x+2}$.

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x+2}$$

$$= \lim_{x\to\infty} \frac{\cancel{x}\sqrt{2+\frac{1}{x^2}}}{\cancel{x}\left(1+\frac{2}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{\sqrt{2+\frac{1}{x^2}\to 0}}{1+\frac{2}{x}\to 0}$$

$$= \boxed{\sqrt{2}}$$

$y = \sqrt{2}$

$$\lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{x+2}$$

$$= \lim_{x\to-\infty} \frac{-\cancel{x}\sqrt{2+\frac{1}{x^2}}}{\cancel{x}\left(1+\frac{2}{x}\right)}$$

$$= \lim_{x\to-\infty} \frac{-\sqrt{2+\frac{1}{x^2}\to 0}}{1+\frac{2}{x}\to 0}$$

$$= \boxed{-\sqrt{2}}$$

$y = -\sqrt{2}$