

2.7: Tangents, Velocities, and Rates of Change

We are now ready to find a formal way of computing the slope of a line tangent to a curve at a point. Re-view the animation from 2.1 posted on my webpage. What happens as the second x coordinate moves closer to the given tangent-line point?

Each secant line above passes through the point $(1, 1)$. If x is the x -coordinate of the second point, write an expression for the slope of the line between the two points.

Write and solve a limit problem which allows us to find the slope of the tangent line at $x = 1$.

More General: Draw any function, a tangent line, and a secant line in the space below. Label the tangent-line point $(a, f(a))$ and the second point on the secant line $(x, f(x))$.

$$m_{sec} =$$

What should happen as the point $(x, f(x))$ moves closer to the tangent-line point $(a, f(a))$? Write a limit which explains this mathematically:

$$m_{tan} =$$

Most General: Draw any function, a tangent line, and a secant line again in the space below. Label the tangent-line point $(a, f(a))$ and let h be the distance between the x values on the secant line. View the new animation posted under today's notes for a visual understanding of this).

$$m_{sec} =$$

$$m_{tan} =$$

The *derivative* of a function at $x = a$ is given by

Examples: Use a limit definition to find the equation of the line tangent to the curve $f(x) = x^2 - 4x + 4$ at the point where $x = 3$.

Use a limit definition to find the derivative of the function $f(x) = \frac{1}{x-2}$. Compute the slopes of the lines tangent to this graph at $x = 0$, $x = 1$, and $x = 3$.

Secant and Tangent Vectors-an Introduction

Illustration of Velocity Vectors (Secant and Tangent):

Example: Find a vector tangent to the curve $\mathbf{r}(t) = (3t^2)\mathbf{i} + \sqrt{t}\mathbf{j}$ at the point $(3,1)$. Then find parametric equations of the line tangent to the curve at this point.