2.7: Tangents, Velocities, and Rates of Change

We are now ready to find a formal way of computing the slope of a line tangent to a curve at a point. Re-view the animation from 2.1 posted on my webpage. What happens as the second $x$ coordinate moves closer to the given tangent-line point?

Each secant line above passes through the point $(1,1)$. If $x$ is the $x$-coordinate of the second point, write an expression for the slope of the line between the two points.

Write and solve a limit problem which allows us to find the slope of the tangent line at $x = 1$.

**More General**: Draw any function, a tangent line, and a secant line in the space below. Label the tangent-line point $(a,f(a))$ and the second point on the secant line $(x,f(x))$.

$m_{sec} =$

What should happen as the point $(x,f(x))$ moves closer to the tangent-line point $(a,f(a))$? Write a limit which explains this mathematically:

$m_{tan} =$
Most General: Draw any function, a tangent line, and a secant line again in the space below. Label the tangent-line point \((a, f(a))\) and let \(h\) be the distance between the \(x\) values on the secant line. View the new animation posted under today’s notes for a visual understanding of this).

\[
m_{sec} =
\]

\[
m_{tan} =
\]

The derivative of a function at \(x = a\) is given by

Examples: Use a limit definition to find the equation of the line tangent to the curve \(f(x) = x^2 - 4x + 4\) at the point where \(x = 3\).
Use a limit definition to find the derivative of the function \( f(x) = \frac{1}{x-2} \). Compute the slopes of the lines tangent to this graph at \( x = 0 \), \( x = 1 \), and \( x = 3 \).

**Secant and Tangent Vectors—an Introduction**

Illustration of Velocity Vectors (Secant and Tangent):
Example: Find a vector tangent to the curve $\mathbf{r}(t) = (3t^2)i + \sqrt{t}j$ at the point (3,1). Then find parametric equations of the line tangent to the curve at this point.