

3.10-Related Rates

Idea: As certain quantities change over time, quantities which are related to them (usually via a formula) also change over time.

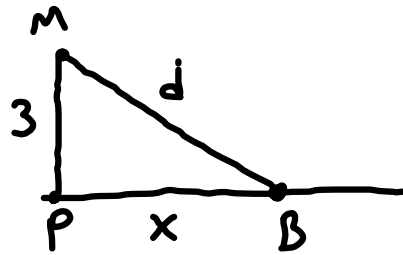
Problem Solving Strategies: (p 217)

- 1) Read entire problem for understanding
- 2) If necessary, draw a diagram
- 3) Label each constant and each variable (changing)
- 4) Find a formula relating your constants/variables
- 5) Write down all given values and rates of variables
- 6) If necessary, eliminate unknown variables by relating them to known variables.
- 7) Differentiate w/r/t time, then subst and solve.

Simultaneous?

Examples:

A man sitting on a pier 3 meters ^{constant} above water pulls on a rope attached at water level to a boat at the rate of 0.5 meters per second. At what rate is the boat approaching the pier when 5 meters ^{snapshot} of rope remain?



$$\frac{dd}{dt} = 0.5$$

$$\frac{dx}{dt} = ?$$

$$d = 5$$

$$3^2 + x^2 = 5^2$$

$$x = 4$$

Pythagoras

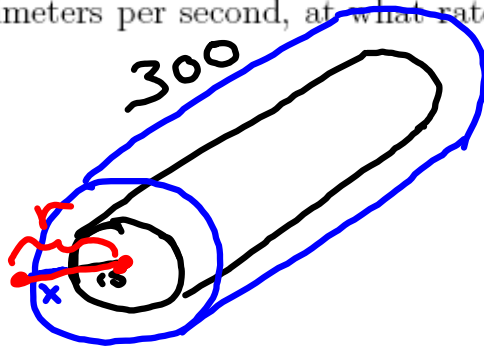
$$3^2 + x^2 = d^2$$

$$0 + 2x \frac{dx}{dt} = 2d \frac{dd}{dt}$$

$$2(4) \frac{dx}{dt} = 2(5) \left(\frac{1}{2}\right)$$

$$\boxed{\frac{dx}{dt} = \frac{5}{8} \frac{m}{s}}$$

A pipe 3 meters long with radius 15 cm is covered with a layer of ice. If the ice is melting at 12 cubic centimeters per second, at what rate is the thickness of the ice decreasing when the ice is 10 cm thick?



$$\frac{dV}{dt} = -12$$

$$\frac{dx}{dt} = ?$$

$$x = 10$$

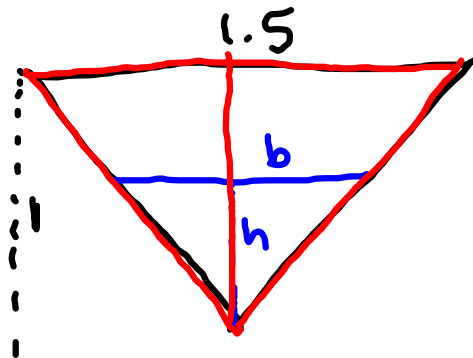
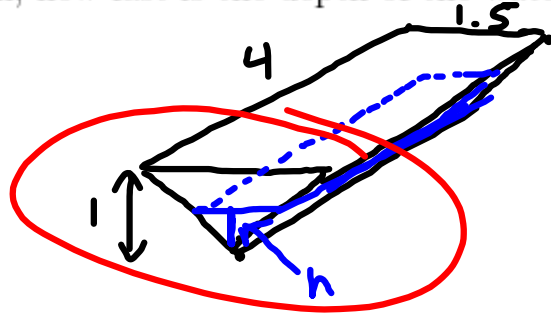
$$V = \pi (15 + x)^2 (300)$$

$$\frac{dV}{dt} = 2\pi (15 + x) \left(\frac{dx}{dt} \right) (300)$$

$$-12 = 600\pi (15 + 10) \frac{dx}{dt}$$

$$\boxed{\frac{dx}{dt} = \frac{-12}{600\pi(25)} \frac{\text{cm}}{\text{sec}}}$$

A feed trough 4 meters long has a cross section that is an isosceles triangle with a base of 1.5 meters long at the top and a height of 1 meter. If water pours into the trough at a rate of $0.25 \text{ m}^3/\text{min}$, how fast is the depth of the water changing when the depth is 0.4 meters?



Similar triangles - use to eliminate b

$$\frac{b}{h} = \frac{1.5}{1}$$

$$b = 1.5h$$

$$\frac{dV}{dt} = \frac{1}{4}$$

$$\frac{dh}{dt} = ?$$

$$h = \frac{2}{5}$$

$$V = \frac{1}{2}bh \cdot 4$$

$$V = 2bh$$

$$V = 2(1.5h)h$$

$$V = 3h^2$$

$$\frac{dV}{dt} = 6h \frac{dh}{dt}$$

$$\frac{1}{4} = 6 \cdot \frac{2}{5} \frac{dh}{dt}$$

$$\frac{5}{2} \cdot \frac{1}{4} = \frac{12}{5} \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{5}{48} \frac{\text{m}}{\text{min}}$$