3.11-Linear Approximation and Differentials

Purpose: To understand differentials and linear approximations to a function near a certain point.

Recall: Given $y = f(x)$, the tangent line is the best approximation to the graph "near" $x = a$.

Why?

Formula for the tangent line:

Therefore, if we want to approximate values of f near a given x-value (a), we can use the tangent line to obtain these approximations.

Example:

Use the linear approximation at $x = \frac{9}{4}$ to estimate $\sqrt{2}$.

A different view: Suppose instead of wanting to estimate actual function values near a given x, we wish to estimate how much the function values change as x changes, as illustrated in the figure from class:

the *differential* dx represents an independent quantity (a small change in x). Then the *differential* dy is given by:

Example:

Use differentials to estimate the volume of a thin cylindrical shell with inner radius r and height h.

Quadratic Approximation: The *Quadratic Approximation* of f at $x = a$ is

Example:

Find the quadratic approximation of $f(x) = \dfrac{1}{x}$ near $x = 2$

On Your Own: #3, 9, 10, 14, 23, 27, 35, 36, 38, 39, 41, 45