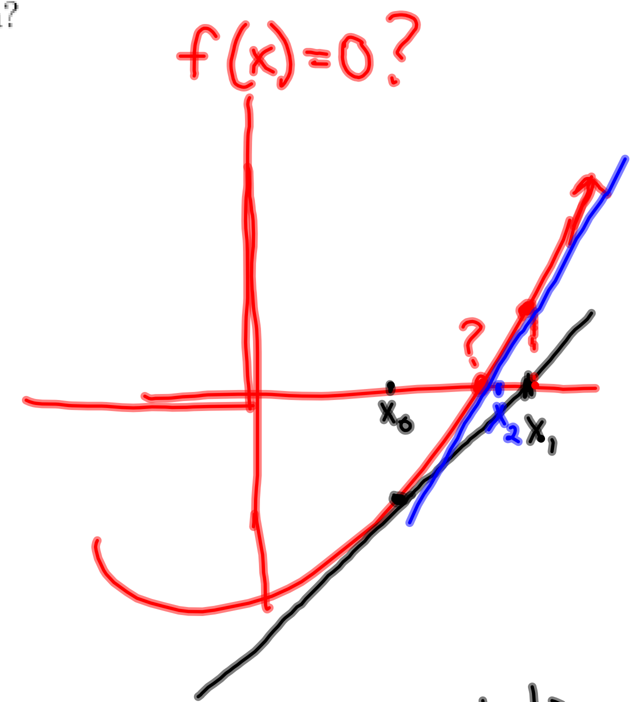
3.12-Newton's Method

Recall: The *Linear Approximation* of f at x_0 is given by $L(x) = f(x_0) + f'(x_0)(x - x_0)$

Set this equal to 0 and solve for x. What do you obtain?



$$f(x_0) + f'(x_0)(x - x_0) = 0$$
$$-f(x_0) \qquad\qquad -f(x_0)$$

$$\frac{f'(x_0)(x - x_0)}{f'(x_0)} = \frac{-f(x_0)}{f'(x_0)}$$

$$x - x_0 = \frac{-f(x_0)}{f'(x_0)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} \quad , \ldots ,$$

x_n's converge to solution

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Example: Given $x_0 = 1$ is an approximate solution of $x^2 = 2$, find the solution to 4 decimal places using Newton's Method.

$$f(x) = x^2 - 2 = 0\,?$$

$$f'(x) = 2x$$

$$x_0 = 1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{-1}{2} = \frac{3}{2}$$

STOP HERE ON QUIZ/EXAM

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{3}{2} - \frac{1/4}{3} = \frac{3}{2} - \frac{1}{12} = \frac{17}{12} \approx 1.4166666\ldots$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = x_2 - \frac{(x_2^2 - 2)}{2x_2} \approx 1.\underline{4142}156$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.\underline{4142}1356$$

$$\boxed{\sqrt{2} \approx 1.4142}$$