

3.2-Derivative Rules

Derivative Rules:

If f and g are differentiable functions, then...

Power Rule $\frac{d}{dx}(x^n) = n x^{n-1}$
"Derivative"

$$\frac{d}{dx}(x^2) = 2x^{2-1} = 2x$$

$$\frac{d}{dx}(cf(x)) = c f'(x)$$

$$\frac{d}{dx}(3x^2) = 3 \cdot 2x = 6x$$

$$\frac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$$

$$\frac{d}{dx}(x^2 + x^3) = 2x + 3x^2$$

Examples: Use a limit definition to find the equation of the line tangent to the curve $f(x) = x^2 - 4x + 4$ at the point where $x = 3$.

Product Rule

$$\frac{d}{dx}(f(x) \cdot g(x)) =$$

~~$f'(x)g(x)$~~

$f(x)g'(x) + g(x)f'(x)$

$$f'(x) = 2x - 4 + 0$$
$$f'(3) = 2 \cdot 3 - 4 = \boxed{2}$$

Quotient Rule

$$\frac{d}{dx} \left(\frac{f(x)}{g(x)} \right) = \frac{g(x) f'(x) - f(x) g'(x)}{[g(x)]^2}$$

Examples:

Compute the derivative of the following:

$$f(x) = x^3 - \frac{5}{4}x^2 + \sqrt{2}x - \frac{3}{7}$$

$$f'(x) = 3x^2 - \frac{5}{2}x + \sqrt{2}$$

$$y = (3x - 2)(2x^2 - 5x + 1)$$

Method I Product Rule

$$y' = (3x - 2)(4x - 5) + (2x^2 - 5x + 1)(3)$$

Method II Multiply out first

$$y = 6x^3 - 15x^2 + 3x - 4x^2 + 10x - 2$$

$$y = 6x^3 - 19x^2 + 13x - 2$$

$$y' = 18x^2 - 38x + 13$$

Find the equation of the line tangent to $f(x) = \frac{3x}{x^2 + 4}$ at the point where $x = 1$.

$$f'(x) = \frac{(x^2+4)(3) - 3x \cdot 2x}{(x^2+4)^2}$$

$$\text{Slope: } f'(1) = \frac{(1+4)(3) - 3 \cdot 2}{(1+4)^2} = \frac{15-6}{25} = \frac{9}{25}$$

$$\text{Point: } f(1) = \frac{3}{1^2+4} = \frac{3}{5}$$

$$\text{Point-Slope } \boxed{y - \frac{3}{5} = \frac{9}{25}(x-1)}$$

$$\text{OR } y = mx + b \\ \frac{3}{5} = \frac{9}{25} \cdot 1 + b \\ \vdots$$

The function $f(x) = \begin{cases} x^2 - 4 & \text{if } x < 4 \\ -2x + 20 & \text{if } x \geq 4 \end{cases}$
 is continuous at $x = 4$. Is the function differentiable at $x = 4$?

$$f'(x) = \begin{cases} 2x & \text{if } x < 4 \\ -2 & \text{if } x > 4 \end{cases}$$

Look at $x=4$: does $\lim_{x \rightarrow 4} f'(x)$ exist?

$$\lim_{x \rightarrow 4^-} f'(x) = 2 \cdot 4 = 8$$

$$\lim_{x \rightarrow 4^+} f'(x) = -2$$

WARNING:

$$f(x) = \begin{cases} 6x & \text{if } x \geq 1 \\ 3x^2 + 2 & \text{if } x < 1 \end{cases}$$

$$f'(x) = \begin{cases} 6 & \text{if } x > 1 \\ 6x & \text{if } x < 1 \end{cases}$$

$\therefore f'(4)$ DNE
 f is not diff at $x=4$

$$\lim_{x \rightarrow 1^-} f(x) = 6 \cdot 1 = 6$$

$$\lim_{x \rightarrow 1^+} f(x) = 6$$

Why $f'(1)$ DNE on Tuesday?

f is NOT cts!

$$\lim_{x \rightarrow 1^-} f(x) = 3 + 2 = 5$$

$$\lim_{x \rightarrow 1^+} f(x) = 6 \cdot 1 = 6$$