

3.3-Rates of Change

Recall: The derivative of a function can measure:

Position/Velocity/Acceleration

de velocity / dt

Example:

A particle moves in a line according to the function $s = f(t) = t^3 + 3t^2 - 24t + 5$, where t is in seconds and s is in feet.

- Find the velocity at time t
- What is the position and velocity after 3 seconds?
- When is the particle at rest?
- Find the total distance traveled in the first 3 seconds.

a) $v = f'(t) = \boxed{3t^2 + 6t - 24}$

b) position: $f(3) = 3^3 + 3 \cdot 3^2 - 24(3) + 5$
 $= 27 + 27 - 72 + 5 = \boxed{-13 \text{ feet}}$

velocity: $f'(3) = 3 \cdot 3^2 + 6 \cdot 3 - 24$
 $27 + 18 - 24 = \boxed{21 \frac{\text{ft}}{\text{s}}}$

c) at rest: $v = f'(t) = 0$

$$3t^2 + 6t - 24 = 0$$

$$3(t^2 + 2t - 8) = 0$$

$$3(t+4)(t-2) = 0$$

$$\boxed{t = -4, t = 2 \text{ seconds}}$$

changed
direction

d) NOT $|f(3) - f(0)|$

$$f(2) = 2^3 + 3 \cdot 2^2 - 24(2) + 5$$

$$= 8 + 12 - 48 + 5$$

$$= -23$$

$$f(0) = 5, f(3) = -13$$

$$|f(2) - f(0)| = |-23 - 5| = 28 \text{ feet}$$

$$|f(3) - f(2)| = |-13 - (-23)| = 10 \text{ feet}$$

$$\text{Total} = \boxed{38 \text{ feet}}$$