

3.4: Limits and Derivatives of Trig Functions

Key Limit: $\lim_{x \rightarrow 0} \frac{\sin x}{x} =$

”Proof”: (zoom in on graph of $y = \sin x$ at $x = 0$)

Key Limit: $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} =$

Proof:

We can use these limits to find the derivative of $f(x) = \sin x$ using the definition:

Similarly, we can show that $\frac{d}{dx}(\cos x) =$

Once we know these, we can find the derivatives of all the other trig functions using quotient rules:

Example: $\frac{d}{dx}(\tan x) =$

Other derivatives:

Examples:

Compute $\lim_{x \rightarrow 0} \frac{2x}{\tan 5x}$

Differentiate $f(x) = \frac{1 - \cos x}{\sin x}$ and $g(x) = \frac{\sin x}{1 + \cos x}$

Determine when the graph of $y = \sec x \tan x$, $0 \leq x \leq 2\pi$ has a horizontal tangent line

On Your Own: #5, 7, 11, 15, 19, 21, 23, 27, 31, 33, 45