3.6-Implicit Differentiation

The equation $F(x, y) = 0$ *implicitly* defines a relation (not necessarily a function) between y and x. The *graph* of $F(x, y) = 0$ is the set of all points (x, y) such that the equation holds ($\{(x, y)|F(x, y) = 0\}$). Given a graph of an implicitly-defined relation, we can still talk about the slope of the line tangent to the curve at a given point.

Method for Implicit Differentiation:

1. Done when y is not explicitly defined as a function of x.
2. Differentiate both sides of the equation, remembering that y depends on x (can call it $y(x)$)
3. Solve for $y'(x)$

Examples:

Find $\frac{dy}{dx}$ implicitly if $x^2y + 2x = 9y + 4$. Then solve for y and show you get the same answer.

Find $\frac{dy}{dx}$ if $x^2y^2 = 2(x^2 + y^2)$

Find the slope of the line tangent to $\sec(x+y) - \tan(x-y) = 1$ at the point (π, π)

Show that the curves $x^2 + y^2 = 9$ and $y = \sqrt{2}\,x$ are orthogonal.

On Your Own: #1, 5, 11, 13, 21, 25, 27, 33, 35, 39, 45