3.6-Implicit Differentiation

The equation $F(x, y) = 0$ implicitly defines a relation (not necessarily a function) between $y$ and $x$. The graph of $F(x, y) = 0$ is the set of all points $(x, y)$ such that the equation holds ($\{(x, y)|F(x, y) = 0\}$). Given a graph of an implicitly-defined relation, we can still talk about the slope of the line tangent to the curve at a given point.

**Method for Implicit Differentiation:**
1. Done when $y$ is not explicitly defined as a function of $x$.
2. Differentiate both sides of the equation, remembering that $y$ depends on $x$ (can call it $y(x)$).
3. Solve for $y'(x)$.

**Examples:**
Find $\frac{dy}{dx}$ implicitly if $x^2y + 2x = 9y + 4$. Then solve for $y$ and show you get the same answer.

Find $\frac{dy}{dx}$ if $x^2y^2 = 2(x^2 + y^2)$.
Find the slope of the line tangent to $\sec(x + y) - \tan(x - y) = 1$ at the point $(\pi, \pi)$

Show that the curves $x^2 + y^2 = 9$ and $y = \sqrt{2}x$ are orthogonal.

**On Your Own:** #1, 5, 11, 13, 21, 25, 27, 33, 35, 39, 45