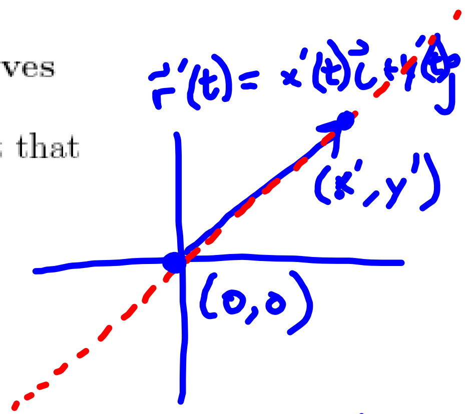
3.9-Slopes and Tangents of Parametrized Curves

To find the slope of the tangent line for a parametrized curve, use the fact that

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \boxed{\frac{\frac{dy}{dt}}{\frac{dx}{dt}}}$$



$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y' - 0}{x' - 0}$$

Examples:

Find an equation of the line tangent to the curve given by $x = 2\cos t$, $y = 1 + \cos(2t)$ at the point where $t = \frac{\pi}{3}$

Slope: $$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cancel{-2}\sin(2t)}{\cancel{-2}\sin t}$$

$$m = \frac{\sin\frac{2\pi}{3}}{\sin\pi/3} = \frac{\sqrt{3}/2}{\sqrt{3}/2} = 1$$

Alt: $$\frac{2\cancel{\sin t}\cos t}{\cancel{\sin t}}$$

$$2\cos\frac{\pi}{3} = 2\cdot\frac{1}{2} = 1$$

Point: $t = \frac{\pi}{3}$, $x = 2\cos\frac{\pi}{3} = 1$

$$y = 1 + \cos\left(\frac{2\pi}{3}\right) = \frac{1}{2}$$

$\quad -\frac{1}{2}$

Equation $\boxed{y - \frac{1}{2} = 1(x - 1)}$ OR $\boxed{y = x - \frac{1}{2}}$

Find an equation of the line tangent to the curve given by $x = t^2 + 2t$, $y = t^3 - t$ at the point $(3, 0)$.

Slope:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2 - 1}{2t + 2}$$

$$m = \frac{3-1}{2+2} = \frac{2}{4} = \frac{1}{2}$$

Find t:

$x = t^2 + 2t = 3$

$t^2 + 2t - 3 = 0$

$(t+3)(t-1) = 0$

$t = -3, t = 1$

$y = t^3 - t = 0$

$t(t^2 - 1) = 0$

$t(t+1)(t-1) = 0$

$t = 0, t = -1, t = 1$

BOTH

Point: $t=1$

$x = 1^2 + 2 \cdot 1 = 3$

$y = 1^3 - 1 = 0$

GIVEN! $(3, 0)$

Equation: $\boxed{y - 0 = \frac{1}{2}(x - 3)}$ OR $\boxed{y = \frac{1}{2}x - \frac{3}{2}}$

Find the points on the curve $x = 4t - t^2$, $y = 1 + t^2$ where the tangent line is horizontal or vertical

Horizontal:

$$\frac{dy}{dx} = 0 \qquad \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 0 \qquad \frac{dy}{dt} = 0$$

$$2t = 0$$

$$t = 0 \quad \text{Find } x \text{ and } y$$

$$x = 4(0) - 0^2 = 0$$

$$y = 1 + 0^2 = 1$$

$\boxed{(0,1)}$ Horiz

Vertical:

$\frac{dy}{dx}$ undef $\qquad \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ undef when $\frac{dx}{dt} = 0$

$$4 - 2t = 0$$

$$t = 2 \quad \text{Find } x \text{ and } y$$

$$x = 4(2) - 2^2 = 4$$

$$y = 1 + 2^2 = 5$$

$\boxed{(4,5)}$ Vert.