4.3-Logarithmic Functions

Recall:

Definition and Properties of Logarithms Define logarithm as inverse of exponential

$y = \log_a x$ means $a^{y} = x$ What exponent?

Properties: 1) $\log_a(xy) = \log_a x + \log_a y$

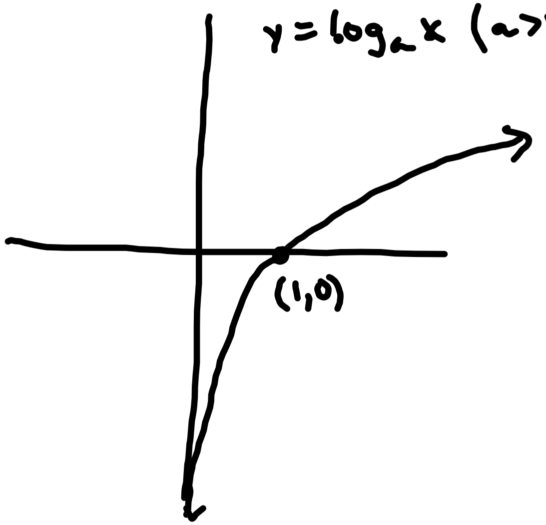
2) $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$

3) $\log_a(x^n) = n \log_a x$

Note: $\log_a(x \pm y)$ DOES NOT CHANGE!

Graphs of Logarithmic Functions:

$y = \log_a x \ (a > 1)$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Intercept: $(1,0)$ x-int

Asymptote: Vertical $x=0$

$\lim_{x \to 0^+} \log_a x = -\infty$

$\lim_{x \to \infty} \log_a x = \infty$

Change of Base Formula: $\log_a x = \dfrac{\ln x}{\ln a}$

Examples:

Calculate $\log_2 \frac{1}{8} = y$ means

$2^y = \frac{1}{8}$ we know $2^3 = 8$

$\boxed{y = -3}$

Solve for a: $\log_8 a = \frac{2}{3}$ means

$8^{2/3} = a$

$a = \left(\sqrt[3]{8}\right)^2$

$= 2^2 = \boxed{4}$

Use properties of logarithms to rewrite $\log_b \frac{y^2\sqrt{z}}{x}$ in terms of $\log_b x$, $\log_b y$, and $\log_b z$

$$= \log_b (y^2 \cdot \sqrt{z}) - \log_b x$$

$$= \log_b y^2 + \log_b \sqrt{z} - \log_b x$$

$$= \boxed{2\log_b y + \tfrac{1}{2}\log_b z - \log_b x}$$

Solve for x: $\ln(3x + 2) - \ln(2x - 3) = 0$

Alt: $\log_e(\quad) = 0$ means $e^0 = (\quad)$

$$\cancel{e}\ \ln\left(\frac{3x+2}{2x-3}\right) = 0$$

$$e$$

$$\frac{3x+2}{2x-3} = 1$$

$$3x + 2 = 2x - 3$$

$$\boxed{x = -5}$$

Check: $\ln(3 \cdot {}^{-}5 + 2)$

-13 Not in domain

$$\boxed{\text{NO SOLUTION}}$$

$$\lim_{x\to\infty} \ln\left(\frac{e^{2x}+2e^{x}}{2e^{2x}+e^{x}}\right)$$ Use the fact that ln is cts

$$= \ln\left(\lim_{x\to\infty} \frac{e^{2x}+2e^{x}}{2e^{2x}+e^{x}}\right)$$ Factor out dominating term e^{2x}

$$\frac{e^{x}}{e^{2x}} = e^{x-2x}$$

$$= \ln\left(\lim_{x\to\infty} \frac{\cancel{e^{2x}}(1+2e^{-x})}{\cancel{e^{2x}}(2+e^{-x})}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{1+2e^{-x}}{2+e^{-x}}\right)$$

$$= \boxed{\ln\left(\frac{1}{2}\right) = -\ln 2}$$

NOTE: $\lim_{x\to-\infty} \ln\left(\frac{e^{2x}+2e^{x}}{2e^{2x}+e^{x}}\right) = \ln 2$