## 4.4-Derivatives of Logarithmic Functions

Why do we know that the function $g(x) = \ln x$ is differentiable?

inverse of $f(x) = e^x$.

$y = \ln x$ means

$x = e^y$ (find $y'$ implicitly)

$1 = e^y \cdot y'$

$y' = \frac{1}{e^y} = \frac{1}{x}$

$$* \quad \frac{d}{dx}(\ln x) = \frac{1}{x} \quad *$$

Alternatively

$$g'(x) = \frac{1}{f'(g(x))} \qquad f'(x) = e^x$$

$$= \frac{1}{e^{\ln x}} = \frac{1}{x}$$

Other Bases:

**Logarithms** $f(x) = \log_a x = \dfrac{\ln x}{\ln a} = \dfrac{1}{\ln a} \cdot \ln x$

So $f'(x) = \dfrac{1}{\ln a} \cdot \dfrac{1}{x}$

$$\boxed{\frac{d}{dx}(\log_a x) = \frac{1}{\ln a} \cdot \frac{1}{x}}$$

**Exponentials** $f(x) = a^x$

$f'(x) = \underset{f'(0)}{K} \cdot a^x$ (See 4.1 notes)

LAST EXAMPLE

Logarithmic Differentiation:

1. Use to differentiate a) lots of products/quotients/powers (optional) or b) $f(x)^{g(x)}$ (required)
2. Take ln of both sides $y = \_\_\_\_$
3. Differentiate implicitly

*Examples:*

Domain $x<0$

Compute and simplify: $\frac{d}{dx}(\ln(-x))$ Chain Rule

$$= \frac{1}{-x} \cdot (-1)$$

Domain $x>0$

$$= \frac{1}{x} \quad \text{Same as } \frac{d}{dx}(\ln x)?$$

Combine the functions

$$\boxed{\frac{d}{dx}(\ln|x|) = \frac{1}{x}}$$

Compute $\frac{d}{dx}(\ln|x^2+3x-5|)$

$$= \frac{1}{x^2+3x-5} \cdot 2x+3$$

$$= \frac{2x+3}{x^2+3x-5} \quad \frac{u'}{u}$$

Product Rule

Compute $f'$ if $f(x) = x^2\ln(3x)$

$$f'(x) = 2x\ln(3x) + x^2 \cdot \frac{\cancel{3}}{\cancel{3}x}$$

$$= \boxed{2x\ln(3x) + x}$$

why $\frac{d}{dx}(\ln 3x) = \frac{1}{x}$?

$$\ln(3x) = \ln 3 + \ln x$$

In 4.1 we proved that, if $f(x) = a^x$, then $f'(x) = Ka^x$, where $K = f'(0)$. Use logarithmic differentiation to find $K$.

$$\ln y = \ln a^x$$

$$\ln y = x \cdot \ln a \quad \text{(Log Property)}$$

$$\frac{y'}{y} = 1 \cdot \ln a$$

$$y' = (\ln a)\, y$$

$$y' = (\ln a)\, a^x$$

$$\boxed{\frac{d}{dx}\left(a^x\right) = (\ln a) \cdot a^x}$$

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