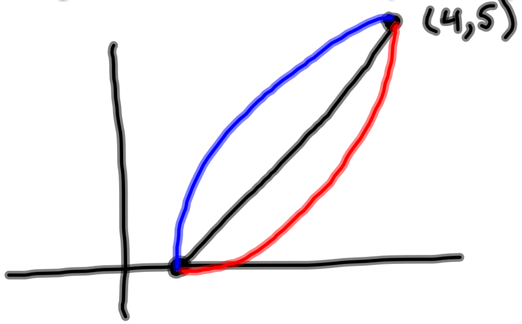
5.1-What Does f' say about f?

Graphical Interpretations of f':

If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on (a,b)
pos.

If $f'(x) < 0$ for all $x \in (a, b)$, then f is decreasing on (a,b)
neg

Example: Draw a function f from $(1, 0)$ to $(4, 5)$ with $f' > 0$.



Concavity ("bend" in graph)

concave up

zero concavity

concave down

Definitions:

a differentiable function f is *concave up* on an interval (a, b) if and only if f' increasing on (a,b)

a differentiable function f is *concave down* on an interval (a, b) if and only if f' decreasing on (a,b)

Therefore...

If $f''(x) > 0$ for all $x \in (a, b)$, then (f' is inc) $\rightarrow$ f is conc up   + + $\cup$

pos

If $f''(x) < 0$ for all $x \in (a, b)$, then (f' is dec) $\rightarrow$ f is conc down   − − $\cap$

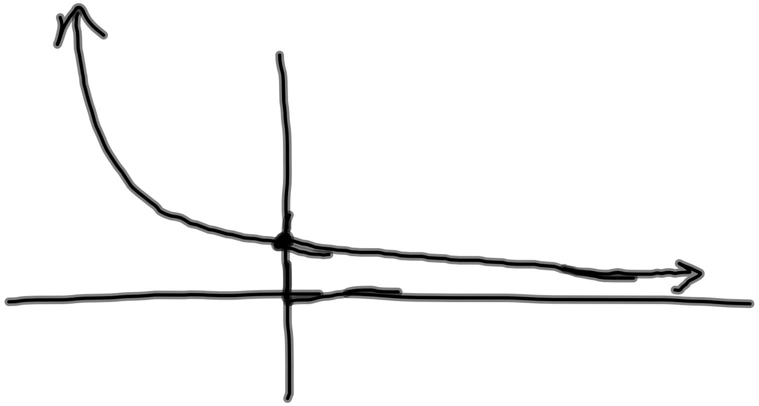
neg

Examples:

pos neg pos

True or False? There exists a function f such that $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x. If true, sketch it; if false, explain why not

(above x axis) (f dec) (f conc up)



$$f(x) = e^{-x} > 0 \text{ for all } x$$

$$f'(x) = -e^{-x} < 0 \text{ for all } x$$

$$f''(x) = +e^{-x} > 0 \text{ for all } x$$

Sketch the graph of a continuous function f on $[0, \infty)$ which satisfies the following:

$\cancel{f(0) = 0,\ f(2) = 2,\ f(3) = 1}$

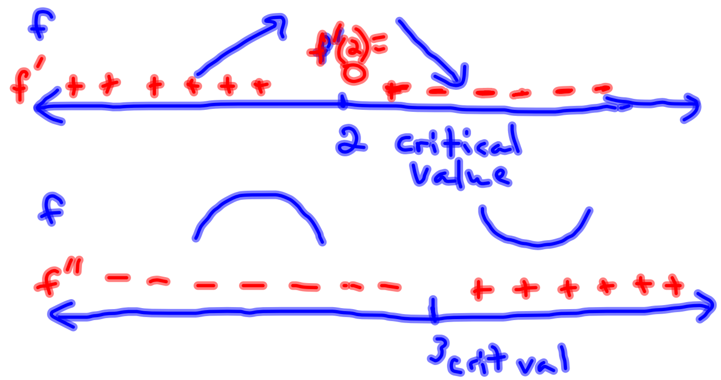
$\cancel{f'(x) > 0 \text{ if } x \in (0, 2)}$ f inc

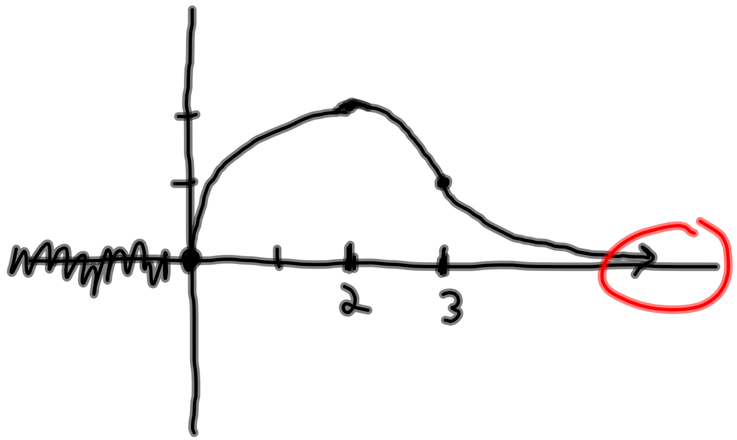
$f'(x) < 0$ if $x \in (2, \infty)$ f dec

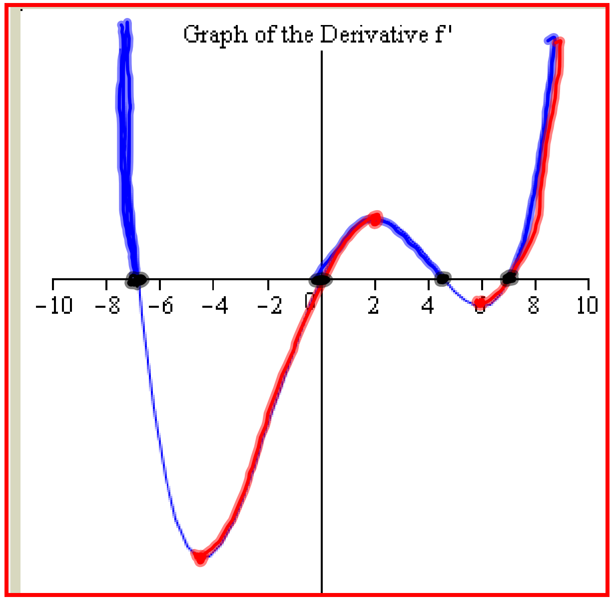
$\cancel{f''(x) > 0 \text{ if } x \in (3, \infty)}$

$\cancel{f''(x) < 0 \text{ if } x \in (0, 3)}$

$\cancel{\lim_{x \to \infty} f(x) = 0}$







On what interval(s) is f increasing?

$f' > 0$

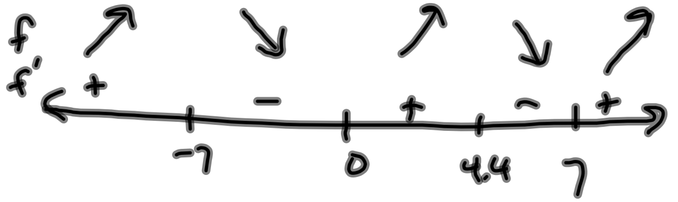
On what interval(s) is f concave up?

$f'' > 0$

f' inc

f ?

crit vals :



BONUS EXAMPLES: Maplets on "Properties of the Graph..."

Mar 30-2:54 PM