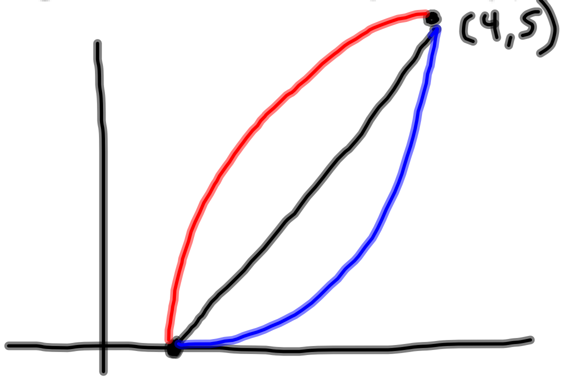
5.1-What Does f' say about f?

Graphical Interpretations of f':

If $f'(x) > 0$ for all $x \in (a, b)$, then f is increasing on (a, b)
pos.

If $f'(x) < 0$ for all $x \in (a, b)$, then f is decreasing on (a, b)
neg

Example: Draw a function f from $(1, 0)$ to $(4, 5)$ with $f' > 0$.



Concavity ("bend" in graph)

concave down

zero concavity

concave up

Definitions:

a differentiable function f is *concave up* on an interval (a, b) if and only if f ' increasing on (a,b)

a differentiable function f is *concave down* on an interval (a, b) if and only if f ' is decreasing on (a,b)

Therefore...

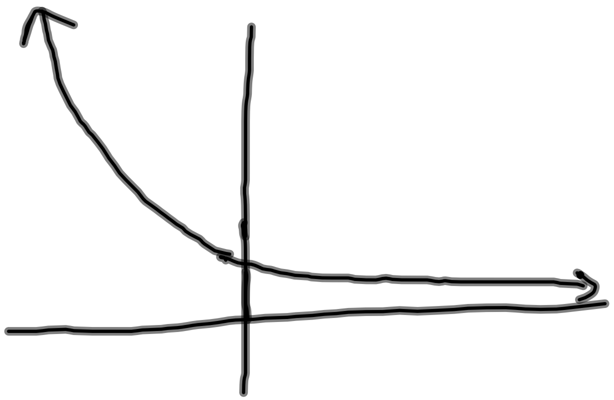
If $f''(x) > 0$ for all $x \in (a, b)$, then (f ' inc) → f conc up + + ∪

If $f''(x) < 0$ for all $x \in (a, b)$, then (f ' dec) → f conc down − − ∩

Examples:

True or False? There exists a function f such that $\overset{\text{pos}}{f(x) > 0}$, $\overset{\text{neg}}{f'(x) < 0}$, and $\overset{\text{pos}}{f''(x) > 0}$ for all x. If true, sketch it; if false, explain why not

(above x-axis) (f dec) (f conc up)



$$f(x) = e^{-x} > 0$$

$$f'(x) = -e^{-x} < 0$$

$$f''(x) = +e^{-x} > 0$$

Sketch the graph of a continuous function f on $[0,\infty)$ which satisfies the following:

~~$f(0)=0,\ f(2)=2,\ f(3)=1$~~

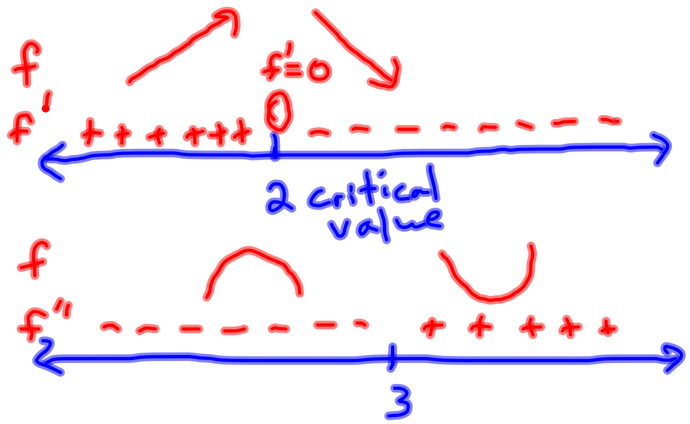
~~$f'(x)>0$ if $x\in(0,2)$~~ pos

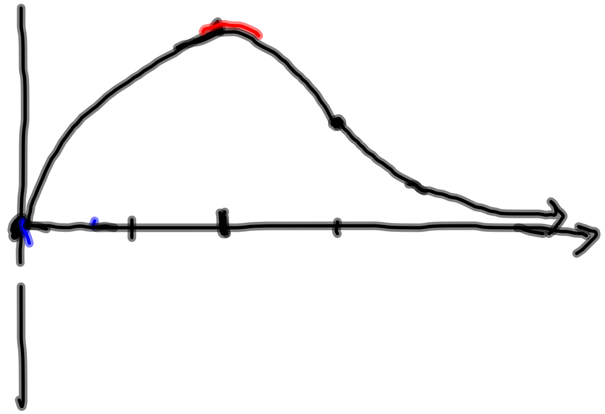
~~$f'(x)<0$ if $x\in(2,\infty)$~~

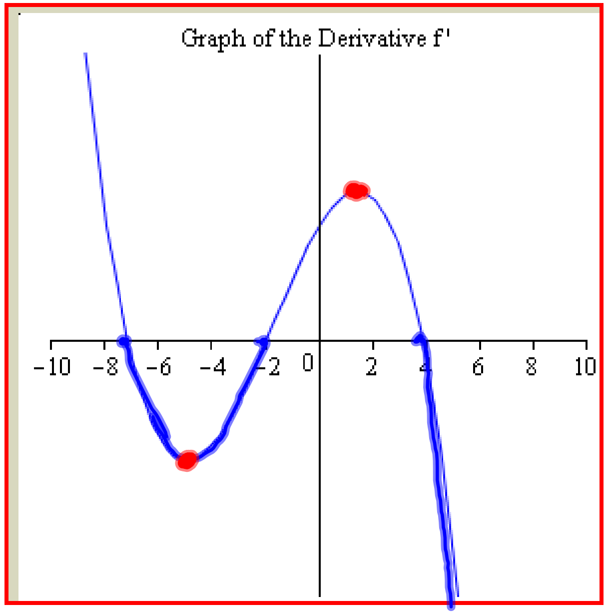
~~$f''(x)>0$ if $x\in(3,\infty)$~~

~~$f''(x)<0$ if $x\in(0,3)$~~

$\lim\limits_{x\to\infty} f(x)=0$ H.A.







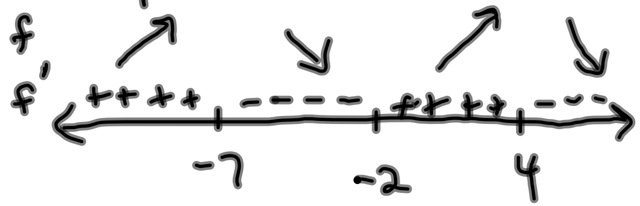
On what interval(s) is f decreasing?

$f' < 0$

Give the x-coordinate(s) of all inflection point(s) of f.

f changes concavity
f' changes direction (min/max)
f" changes sign

Graph of f? critvals ($f'=0$)



BONUS EXAMPLES: Maplets on "Properties of the Graph..."