5.3-Derivatives and the Shapes of Curves

Mean Value Theorem: If f cts on $[a,b]$ and f diff (a,b) then there is a number $c \in [a,b]$ such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

Recall 5.1: What f' and f'' say about f:

$f' > 0 \to f$ inc

$f' < 0 \to f$ dec

$f'' > 0 \to f'$ inc $\to f$ conc up

$f'' < 0 \to f'$ dec $\to f$ conc down

Recall: f has inflection point if $f(a)$ exist and f changes concavity at $x=a$. (if f'' exists, $f''(a)=0$)

Second Derivative Test: if f has a critical value at $x=a$, and:

$\cup$ 1) $f''(a) > 0$, then f has a rel min at $x=a$

$\cap$ 2) $f''(a) < 0$, then f has a rel max at $x=a$

3) $f''(a) = 0$, then ?

Examples:

Determine where the function $x^3 - 3x^2 + 4$ is increasing and decreasing, concave up, and concave down.

inc/dec

$$f'(x) = 3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$x=0 \quad x=2$

No criticals with f' DNE

f

f' + + + + | - - - - - - | + + + + +

0   2

Test $x=-1$: $- \cdot -$

Test $x=1$: $+ \; -$

Test $x=3$: $+ \cdot +$

$\boxed{\begin{array}{l} f \text{ inc } (-\infty,0) \cup (2,\infty) \\ f \text{ dec } (0,2) \end{array}}$

rel max $x=0$

rel min $x=2$

conc

$$f''(x) = 6x - 6 = 0$$

$$x = 1$$

f

f'' - - - - | + + + + +

1

Test $x=0$

Test $x=2$

$\boxed{\begin{array}{l} \text{conc up } (1,\infty) \\ \text{conc down } (-\infty,1) \end{array}}$

infl pt when $x=1$

$(1, \_\_)$

Find the horizontal and vertical asymptotes, intervals of direction, and intervals of concavity for $f(x) = \ln|1 - x^2|$ and sketch the graph.

Asym

Vert Asym: $x = 1, -1$ $(1 - x^2 = 0)$

No Horiz Asym $(f(x) \to \infty$ as $x \to \pm\infty)$

inc/dec

IMPORTANT!

$f'(x) = \dfrac{-2x}{1-x^2} = 0 \; (1-x^2)$

$-2x = 0$

$x = 0$

* f' DNE at $x = 1, -1$ *

f: ↘ ↗ ↘ ↗

f': - - - - | + + + + | - - - - | + + + +

-1 0 1

Test $x=-2$: $\frac{+}{-}$; Test $x=-\frac{1}{2}$: $\frac{+}{+}$; Test $x=\frac{1}{2}$: $\frac{-}{+}$; Test $x=2$: $\frac{-}{-}$

f inc $(-1,0) \cup (2,\infty)$

f dec $(-\infty,-1) \cup (0,1)$

NOTE:
rel max at $(0,0)$
(NO max/min at ± 1 since Vert Asym)

conc

$$\frac{(1-x^2)(-2) - (-2x)(-2x)}{(1-x^2)^2} = 0$$

$-2 - 2x^2$

$-2 + 2x^2 - 4x^2 = 0$

$-2 - 2x^2 = 0$

$-2x^2 = 2$

$x^2 = -1$

NO SOLN

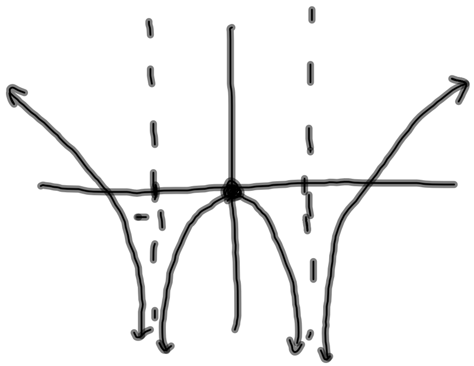
f: ∩ ∩ ∩

f'': - - - | - - - | - - - -

-1 1

$\frac{-}{+}$ for all x

f conc down $(-\infty,\infty)$



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