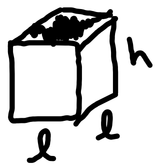
5.5-Applied Max/Min Problems

Goal: To optimize a practical value subject to certain restrictions (often given geometrically)

Examples:

A rectangular box with a square base and no top has volume V. Find the dimensions of the box which minimize its surface area.



Goal: Min $S = \ell^2 + 4\ell h$

Restriction: $\underset{\text{const}}{V} = \ell^2 h$

$h = \frac{V}{\ell^2}$

use to eliminate extra variable

Goal: Min $S = \ell^2 + 4\cancel{\ell}\left(\frac{V}{\ell^{\cancel{2}}}\right)$; $\ell > 0$

$= \ell^2 + \frac{4V}{\ell}$

$S' = \left(2\ell - \frac{4V}{\ell^2}\right)\ell^2 = 0 \cdot \ell^2$

$2\ell^3 - 4V = 0$

$2\ell^3 = 4V$

$\ell^3 = 2V$

$\ell = \sqrt[3]{2V}$

* MUST SHOW MIN!

* How show max/min?

a) If closed bdd interval, apply Extreme Val Thm

b) $\longleftarrow \sqrt[3]{2V} \longrightarrow$

c) Second Der Test

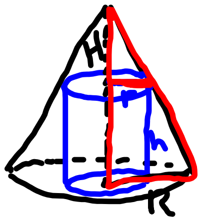
$S'' = 2 + \frac{8V}{\ell^3} > 0$ $\cup$ min

Dimensions: $\boxed{\sqrt[3]{2V} \times \sqrt[3]{2V} \times \frac{V}{\sqrt[3]{4V^2}}}$

A right circular cylinder is inscribed in a right circular cone of radius R and height H. Find the largest possible cylinder.

volume

const



Goal: Max $V=\pi r^2 h$

Restriction: $\frac{H}{R}=\frac{H-h}{r}$ Solve for h

$$Hr = HR - Rh$$

$$Rh = HR - Hr$$

$$h = \frac{HR-Hr}{R} = 0$$ and solve to get upper bound

$$\text{Max } V=\pi r^2\left(\frac{HR-Hr}{R}\right);\quad 0\le r\le R$$

$$V=\frac{\pi r^2 HR - \pi H r^3}{R}$$

$$V'=\frac{2\pi HRr - 3\pi H r^2}{R}=0$$

$$2\pi HRr - 3\pi H r^2=0$$

$$\pi H r(2R-3r)=0$$

$r=0$ $\quad$ $r=\frac{2R}{3}$

$V(0)=0$

$V(R)=0$

$V\left(\frac{2R}{3}\right)>0$ Max

$$\boxed{\text{Max } V = \pi\left(\frac{2R}{3}\right)^2\left(\frac{HR-H\frac{2R}{3}}{R}\right)} = \pi\left(\frac{2R}{3}\right)\left(\frac{H}{3}\right)$$

(x,y)

Find the shortest distance from the point $(10,0)$ to the circle $x^2+y^2=25$.

Goal: Min $d=\sqrt{(x-10)^2+(y-0)^2}$

(same critical value!)

Easier to look at

Min $d^2 = D = (x-10)^2+y^2$

Restriction $x^2+y^2=25$ (point must be on circle)

$y^2=25-x^2$

$=0$ and solve

Min $D=(x-10)^2+25-x^2$; $-5\le x\le 5$

$2(x-10)(1)-2x=0$

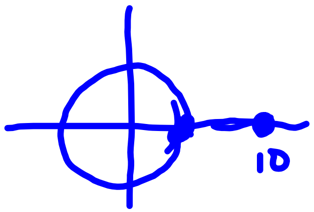
$\cancel{2x}-20-\cancel{2x}=0$

No crit value

Ext Val Thm:

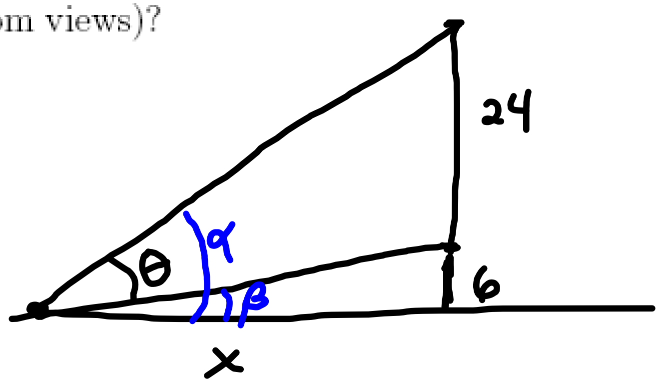
$x=5$ $\quad D=(5-10)^2+25-5^2=25$

$x=-5$ $\quad D=(-5-10)^2+25-(-5)^2=225$



Min when $x=5$ distance $=\sqrt{25}=\boxed{5\text{ units}}$

A movie screen is 24 feet tall and hangs 6 feet above a level floor. How far away from the screen should a person stand in order to maximize their viewing angle (the angle between the top and bottom views)?



Goal: Max $\theta = \tan^{-1}\left(\frac{30}{x}\right) - \tan^{-1}\left(\frac{6}{x}\right)$;

$x > 0$

$$\theta' = \frac{1}{1+\left(\frac{30}{x}\right)^2} \cdot \frac{-30}{x^2} - \frac{1}{1+\left(\frac{6}{x}\right)^2} \cdot \frac{-6}{x^2} = 0$$

$$\frac{-30}{x^2+900} + \frac{6}{x^2+36} = 0$$

$$\frac{6}{x^2+36} = \frac{30}{x^2+900}$$

$$6x^2 + 5400 = 30x^2 + 1080$$

$$4320 = 24x^2$$

$$x^2 = \frac{4320}{24} = \frac{720}{4} = 180$$

$$\boxed{x = \sqrt{180}\ \text{ft}}$$

#line easiest here