# 5.7-Antiderivatives

$F$ is an antiderivative of $f$ if and only if $f(x) = F'(x)$

*Antiderivative Rules:*

| Derivative | Original Function | Derivative | Original Function |
|---|---|---|---|
| $x^n$ ($n \neq -1$) | $\frac{1}{n+1}x^{n+1} + C$ | $\csc^2 x$ | $-\cot x + C$ |
| $f(x) \pm g(x)$ | $F(x) \pm G(x) + C$ | $\csc x \cot x$ | $-\csc x + C$ |
| $c f(x)$ | $c F(x) + C$ | $x(t)\vec{i} + y(t)\vec{j}$ | $X(t)\vec{i} + Y(t)\vec{j} + \vec{C}$ (by component) |
| $c$ | $cx + C$ | $e^x$ | $e^x + C$ |
| $\sin x$ | $-\cos x + C$ | $\frac{1}{x}$ | $\ln\|x\| + C$ |
| $\cos x$ | $\sin x + C$ | $\frac{1}{\sqrt{1-x^2}}$ | $\sin^{-1} x + C$ |
| $\sec^2 x$ | $\tan x + C$ | $\frac{1}{1+x^2}$ | $\tan^{-1} x + C$ |
| $\sec x \tan x$ | $\sec x + C$ | $\frac{1}{x\sqrt{x^2+1}}$ | $\sec^{-1} x + C$ |

*Examples*:

Find the most general antiderivative of $f(t) = \sec^2 t - t^3 + 10$

$$F(t) = \tan t - \frac{1}{4}t^4 + 10t + C$$

$x^{\frac{1}{2}+1}$ $\quad x^{-\frac{1}{2}+1}$ $\quad \cancel{(1-x^2)^{-\frac{1}{2}}}$

Find the most general antiderivative of $f(x) = \sqrt{x} + \dfrac{1}{\sqrt{x}} - \dfrac{1}{\sqrt{1-x^2}}$

$$\boxed{F(x) = \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - \sin^{-1} x + C}$$

Find $f$ if $f'(x) = \dfrac{(x^2-4)^2}{x}$, $f(1) = 0$
(IGNORE for now)

Square numerator $f'(x) = \dfrac{x^4 - 8x^2 + 16}{x}$ $16x^{-1}$

split numerator $f'(x) = x^3 - 8x^1 + \dfrac{16}{x}$

NOTE: CANNOT SPLIT DENOMINATOR!

$f(x) = \frac{1}{4}x^4 - 4x^2 + 16\ln|x| + C.$

Back to our info: $f(1) = 0$
i.e. what is C so the graph passes through (1, 0)?

$0 = \frac{1}{4}\cdot 1^4 - 4(1)^2 + 16\ln|1| + C$

Find C

$0 = \frac{1}{4} - 4 + C$

$0 = \frac{-15}{4} + C$

$C = \frac{15}{4}$

$$\boxed{f(x) = \frac{1}{4}x^4 - 4x^2 + 16\ln|x| + \frac{15}{4}}$$

Apr 14-10:00 AM

$$F(t) = 20\vec{j}$$

A force with magnitude 20 N acts in the positive $y$ direction on an object with mass 4 kg. The object starts at the origin with initial velocity $\mathbf{v}(0) = \mathbf{i} - \mathbf{j}$. Find its position function and speed at any time $t$.

$\frac{F}{m} = \frac{ma}{m}$, so $\vec{a}(t) = 5\vec{j}$ (antiderivative/integrate to find velocity)

$$\vec{v}(t) = 5t\vec{j} + \vec{C}$$

$$\vec{i} - \vec{j} = 5(0)\vec{j} + \vec{C} \rightarrow \vec{C} = \vec{i} - \vec{j}$$

$$C_x\vec{i} + (5t + C_y)\vec{j}$$

$$\vec{v}(t) = 5t\vec{j} + (\vec{i} - \vec{j})$$
$$= 1\vec{i} + (5t-1)\vec{j}$$

can do separate constants in $\vec{i}$ and $\vec{j}$

$$\vec{r}(t) = (t + C_x)\vec{i} + \left(\frac{5}{2}t^2 - t + C_y\right)\vec{j}$$

$$\vec{0} = (0 + C_x)\vec{i} + \left(\frac{5}{2}\cdot 0^2 - 0 + C_y\right)\vec{j}$$

$C_x = 0$

$C_y = 0$

$$\boxed{\vec{r}(t) = t\vec{i} + \left(\frac{5}{2}t^2 - t\right)\vec{j}}$$

$$\text{speed} = |\vec{v}(t)| = \boxed{\sqrt{1^2 + (5t-1)^2}}$$