**6.5-Substitution**

**Recall:** Chain Rule for Derivatives: $\frac{d}{dx}(f(g(x))) = f'(g(x))\,g'(x)$

Therefore $\int f'(g(x))\,g'(x)\,dx = f(g(x)) + C$

**Problem**: Recognizing when you have an integral of this form and what $f$ and $g$ are.

**Solution**: Substitute for $g(x)$, your "inner function"

*Examples*:

in denominator, so DON'T multiply out

Compute $\frac{1}{3}\int_1^2 \frac{3\,dx}{(3x-2)^2}\,dx$

Let $u = 3x-2$ ("stuff" in parentheses)

$du = 3\,dx$

$= \frac{1}{3}\int_1^4 \frac{1}{u^2}\,du$ $\quad u^{-2}$ $\quad *$

$= -\frac{1}{3}u^{-1}\Big|_1^4$

$= -\frac{1}{3}\left(\frac{1}{4}-1\right)$

$= -\frac{1}{3}\left(\frac{-3}{4}\right) = \boxed{\frac{1}{4}}$

3 options for def integrals:

1) do the indef integral, then use to solve def integral

2) explicitly state variable in boundaries $\frac{1}{3}\int_{x=1}^{x=2} \frac{1}{u^2}\,du$

if $x=1$, $u=3\cdot1-2=1$

if $x=2$, $u=3\cdot2-2=4$

$*$ 3) change boundaries to new variable.

Compute $\displaystyle\int \frac{e^x}{e^{2x}+1}\,dx$

Want du = this

~~Let $u = e^{2x}+1$~~

~~$du = 2e^{2x}\,dx$~~

cannot deal with

Instead, let $u = e^x$

$du = e^x\,dx$

$$= \int \frac{e^x}{(e^x)^2+1}\,dx$$

$$= \int \frac{1}{u^2+1}\,du$$

$$= \tan^{-1}(u) + C$$

$$= \boxed{\tan^{-1}(e^x) + C}$$

Compute $\frac{1}{2}\int_0^{\pi/4} \cos^3(2y)\sin(2y)\cdot 2\,dy$

Let $u = 2y$

$du = 2\,dy$

if $y = 0$, $u = 2\cdot 0 = 0$

$y = \frac{\pi}{4}$, $u = 2\cdot\frac{\pi}{4} = \frac{\pi}{2}$

$$= \frac{-1}{2}\int_0^{\pi/2} \underbrace{\cos^3 u}_{(\cos u)^3}(-\sin u\,du)$$

Let $t = \cos u$

$dt = -\sin u\,du$

if $u = 0$, $t = \cos 0 = 1$

$u = \frac{\pi}{2}$, $t = \cos\frac{\pi}{2} = 0$

$$= -\frac{1}{2}\int_1^0 t^3\,dt$$

$$= \frac{1}{2}\int_0^1 t^3\,dt$$

$$= \frac{1}{2}\cdot\frac{1}{4}t^4\Big|_0^1$$

$$= \frac{1}{8}(1-0) = \boxed{\frac{1}{8}}$$

if change boundaries, DO NOT change vars

if DO NOT change boundaries, DO change vars

NOTE: we could have started by letting $u = \cos 2y$

$du = -2\sin 2y$

$(x^2+1)^{1/2}$

Compute $\int x^3\sqrt{x^2+1}\,dx$

Let $u = x^2+1 \to x^2 = u-1$

$du = 2x\,dx$

$$= \frac{1}{2}\int x^2\sqrt{x^2+1}\;2x\,dx$$

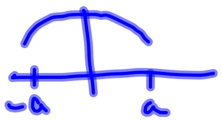
$$= \frac{1}{2}\int x^2\sqrt{u}\;du$$

$$= \frac{1}{2}\int (u-1)\overset{u^{1/2}}{\sqrt{u}}\;du$$

$$= \frac{1}{2}\int \left(u^{3/2} - u^{1/2}\right)du$$

$$= \frac{1}{2}\left(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right) + C$$

$$= \boxed{\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C}$$




**Useful for 152**: If $f$ is even, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

If $f$ is odd, then $\int_{-a}^{a} f(x)\,dx = 0$