MATH 151
FALL 2005

FINAL EXAM
TEST FORM A

NAME ________________________ ________________________
(print) LAST FIRST

SIGNATURE ____

SECTION NUMBER

INSTRUCTIONS

1. In Part I, problems 3-12, circle the correct choice on your exam. Write your answers to problems 1-2 in the space provided. Calculators are not allowed during this part of the exam. Use the back of each page for scratch work. This part will be collected after 1 hour.

2. In Part II (Problems 13-20), write all solutions in the space provided. Calculators are not allowed until Part I is collected, but you may work on this part (without a calculator) until that time. Use the back of each page for scratch work. CLEARLY INDICATE YOUR FINAL ANSWER.

Part IA - Derivatives and Integrals

1. (3 points each) Write the derivative of each of the following. DO NOT SIMPLIFY YOUR ANSWER.

a) $f(x) = \dfrac{x^4 - 2x + 5}{\cos(3x)}$

b) $g(t) = e^{\sin(2t)} + 2\sin(e^t)$

c) $y = \ln(x^3 + 1) + \tan^{-1} x - \dfrac{1}{1 + x^2}$

2. (3 points each) Compute the following integrals:

a) $\displaystyle\int_1^4 (6x^2 - \sqrt{x})\, dx$

b) $\displaystyle\int_0^1 xe^{-x^2}\, dx$

c) $\displaystyle\int (3\cos x + 5\sin x)\, dx$

For problems 3 through 12, circle the correct answer on your exam. Each question is worth 4 points.

3. Let $\mathbf{a} = -2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = \mathbf{i} - 4\mathbf{j}$. Find the scalar projection of $\mathbf{a}$ onto $\mathbf{b}$.

a) $\dfrac{-14}{\sqrt{17}}$ b) $\left\langle \dfrac{-14}{17}, \dfrac{56}{17} \right\rangle$

c) $\dfrac{-14}{\sqrt{13}}$ d) -14 e) $\left\langle \dfrac{28}{17}, \dfrac{-42}{17} \right\rangle$

4. $\displaystyle\lim_{x \to -2} \frac{(x+2)\cos(\pi x)}{x^2 - 4} =$

a) $-\dfrac{1}{4}$ b) $\dfrac{1}{4}$ c) 0

d) DNE e) 1

5. Given $f(x) = \begin{cases} 3 - \frac{1}{2}x & \text{if } x < 4 \\ 1 & \text{if } x = 4 \\ -4 + \frac{3}{2}x & \text{if } x > 4 \end{cases}$
which of the following statements about the continuity of f at $x = 4$ is true?

a) f is continuous only from the right
b) f is continuous only from the left
c) f has a removable discontinuity
d) f is continuous
e) none of the above is true

6. Let f be a differentiable functions with $f(4) = 3$, $f'(4) = 2$. Find the slope of the line tangent to the curve $y = (f(x))^3$ at $x = 4$.

a) 12 b) 36 c) 18

d) 27 e) 54

7. Given the curve $x^3y + 2xy^3 = 12$, find the slope of the tangent line at $(2, 1)$

a) $\frac{10}{7}$ b) 10 c) $-\frac{10}{7}$

d) $-\frac{7}{10}$ e) $-\frac{1}{10}$

8. Find the linear approximation of $f(x) = e^x$ at $x = 1$.

a) $1 + 1(x - 1)$ b) $e + 1(x - 1)$

c) $e + ex$ d) $e + e(x - 1)$

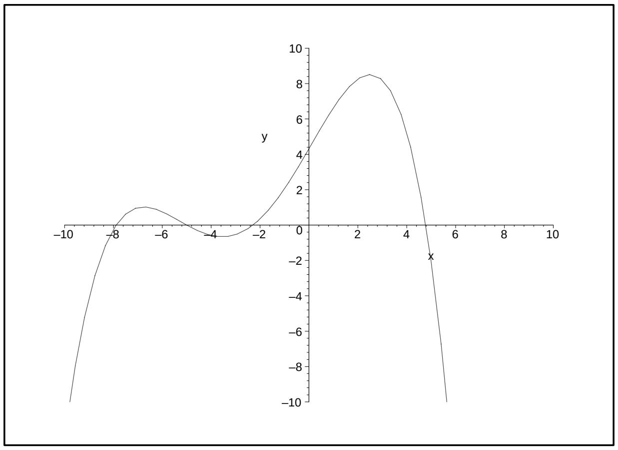
e) $1 + x$

9. $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} =$

a) e b) 0 c) 1

d) e^2 e) 2

10. The graph of THE ORIGINAL FUNCTION f is shown. To the nearest integers, on which interval(s) is $f'(x) > 0$?



a) $(-7, -4) \cup (2, \infty)$

b) $(-\infty, -8) \cup (-5, -2) \cup (5, \infty)$

c) $(-\infty, -7) \cup (-4, 2)$

d) $(-8, -5) \cup (-2, 5)$

e) $(-5, 0)$

11. On which of the following intervals is $f(x) = x^2(1 - x)$ increasing?

a) None of these

b) $(-1, 0)$

c) $(0, 1)$

d) $\left(-\frac{2}{3}, \frac{2}{3}\right)$

e) $\left(0, \frac{2}{3}\right)$

12. After an appropriate substitution, which of the following integrals is equivalent to $\int_2^4 \frac{x}{\sqrt{x^2 - 1}}\,dx$?

a) $\int_3^{15} \frac{2}{\sqrt{u}}\,du$

b) $\int_2^4 \frac{1}{\sqrt{u}}\,du$

c) $\int_3^{15} \frac{1}{\sqrt{u}}\,du$

d) $\int_3^{15} \frac{1}{2\sqrt{u}}\,du$

e) $\int_2^4 \frac{u}{\sqrt{u^2 - 1}}\,du$

Part II - Work Out Problems

Work the following problems in the space provided. Calculators are allowed after 1 hour, but all answers must be algebraically supported to receive full credit. Each problem is worth 7 points.

13. An object is moved with a constant force of 20 Newtons as shown in the diagram below. The object also has a resistance force of 4 Newtons acting as shown. Find the magnitude and direction (angle with the horizontal) of the resultant force.

14. Use the limit definition to find the derivative of $f(x) = 4x^2 - x$.

15. Find the equation (in any form) of the line tangent to the curve $\mathbf{r}(t) = \cos(2t)\mathbf{i} + t\sin t\mathbf{j}$ at the point where $t = \dfrac{\pi}{6}$.

16. An observer sees a hot-air balloon 1 km away rising at a rate of 10 km/hr. How fast is the distance directly between the observer and the balloon changing when the balloon is 3 km above ground?

17. A wire 30cm long is to be cut into at most 2 pieces. The first piece is bent into a square; the second is bent into a circle. How should the wire be divided so that the total area enclosed by the figures is a maximum?

18. Approximate $\int_1^5 \ln x \, dx$ using a partition of $P = \{1, 2, 3, 5\}$ and taking x_i^* to be the midpoint of each subinterval.