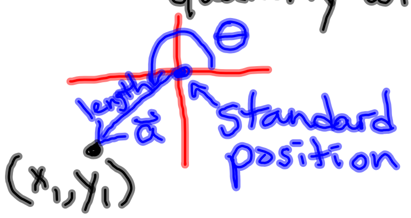
1 1.1: Vectors

Definitions:

vector: A quantity with magnitude and direction



One-to-one correspondance between standard vectors and points in plane

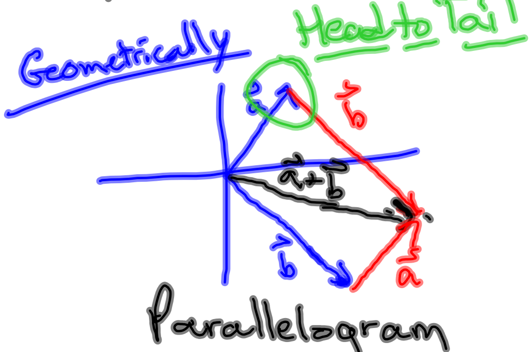
$\vec{a} = \langle x_1, y_1 \rangle$

addition

If $\vec{a} = \langle x_1, y_1 \rangle$ and $\vec{b} = \langle x_2, y_2 \rangle$

then $\vec{a} + \vec{b} = \langle x_1 + x_2, y_1 + y_2 \rangle$

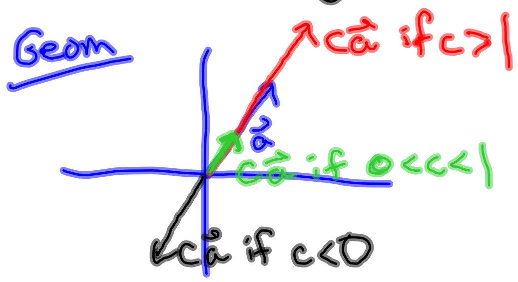
(resultant)



scalar multiplication

If $\vec{a} = \langle x_1, y_1 \rangle$ then

$c\vec{a} = \langle cx_1, cy_1 \rangle$



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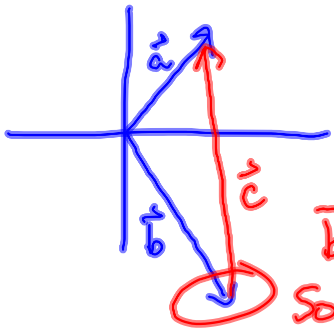
subtraction

If $\vec{a}=\langle x_1,y_1\rangle$ and $\vec{b}=\langle x_2,y_2\rangle$

then $\vec{a}-\vec{b}=\langle x_1-x_2,\ y_1-y_2\rangle$



$\vec{b}+\vec{c}=\vec{a}$

So $\vec{c}=\vec{a}-\vec{b}$

End − Start

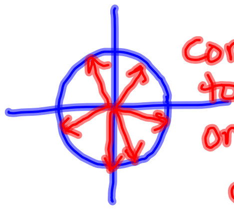
magnitude: if $\vec{a}=\langle x_1,y_1\rangle$ then

$$|\vec{a}|=\sqrt{x_1^2+y_1^2}$$

unit vector:

a vector whose magnitude $=1$

infinite # of unit vectors:



correspond to points on unit circle,

i and **j**:

$\vec{i}=\langle 1,0\rangle$

$\vec{j}=\langle 0,1\rangle$

$$\begin{aligned}\langle x_1,y_1\rangle &= \langle x_1,0\rangle+\langle 0,y_1\rangle\\ &= x_1\langle 1,0\rangle+y_1\langle 0,1\rangle\\ &= x_1\vec{i}+y_1\vec{j}\end{aligned}$$

$3\vec{i}+4\vec{j}=\langle 3,4\rangle$

$\langle 2,-5\rangle=2\vec{i}-5\vec{j}$

Examples:

Given the vectors $\mathbf{a} = <3, -5>$ and $\mathbf{b} = 2\mathbf{i} - 4\mathbf{j}$, write $\mathbf{i} + \mathbf{j}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$\vec{i} + \vec{j} = s\vec{a} + t\vec{b}$$

$$\vec{i} + \vec{j} = s(3\vec{i} - 5\vec{j}) + t(2\vec{i} - 4\vec{j})$$

$$\vec{i} + \vec{j} = (3s\vec{i} - 5s\vec{j}) + (2t\vec{i} - 4t\vec{j})$$

$$1\vec{i} + 1\vec{j} = (3s + 2t)\vec{i} + (-5s - 4t)\vec{j}$$

$$\left.\begin{aligned} 2(1 &= 3s + 2t) \\ 1 &= -5s - 4t \end{aligned}\right\} \text{ solve for } s, t$$

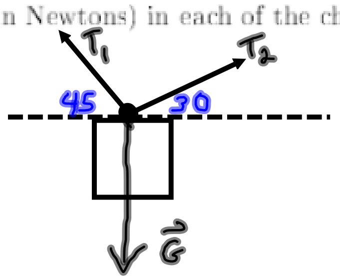
$$\boxed{s = 3,\ t = -4} \quad \text{OR}$$

$$\boxed{\vec{i} + \vec{j} = 3\vec{a} - 4\vec{b}}$$

$$\begin{aligned} 2 &= 6s + 4t \\ 1 &= -5s - 4t \\ \hline 3 &= s \end{aligned}$$

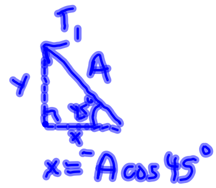
$$1 = -5(3) - 4t$$
$$-15$$
$$16 = -4t$$
$$t = -4$$

A 10 kg sign is to be hung from chords as shown in the diagram given in class. Find the tensions (in Newtons) in each of the chords.



$\sum \vec{F} = \vec{0}$

$(F = ma)$



$\vec{G} = -98\vec{j}$

Let $A = |T_1|$ and $B = |T_2|$

$\vec{T}_1 = (-A\cos 45°)\vec{i} + (A\sin 45°)\vec{j}$
$= \left(-\frac{\sqrt{2}}{2}A\right)\vec{i} + \left(\frac{\sqrt{2}}{2}A\right)\vec{j}$

$\vec{T}_2 = (B\cos 30°)\vec{i} + (B\sin 30°)\vec{j}$
$= \left(\frac{\sqrt{3}}{2}B\right)\vec{i} + \left(\frac{1}{2}B\right)\vec{j}$

$\vec{i}: \frac{\sqrt{3}}{2}B - \frac{\sqrt{2}}{2}A = 0$
$\vec{j}: \frac{1}{2}B + \frac{\sqrt{2}}{2}A - 98 = 0$ } solve

$\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)B - 98 = 0$

$B = \frac{98 \cdot 2}{\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) \cdot 2} = \boxed{\frac{196}{\sqrt{3}+1} N}$ $\approx 71.74N$

$\frac{\sqrt{3}}{\cancel{2}}\left(\frac{196}{\sqrt{3}+1}\right) - \frac{\sqrt{2}}{\cancel{2}}A = 0$

$A = \boxed{\frac{196\sqrt{3}}{(\sqrt{3}+1)\sqrt{2}} N}$ $\approx 87.86N$

On Your Own: Given $\mathbf{a} = < 2, 1 >$ and $\mathbf{b} = 6\mathbf{i} + 7\mathbf{j}$, find each of the following: $\mathbf{a} + \mathbf{b}$, $5\mathbf{a} - 2\mathbf{b}$, and a unit vector in the direction of $\mathbf{b}$.

$$\vec{a} + \vec{b} = (2\vec{i} + \vec{j}) + (6\vec{i} + 7\vec{j}) = \boxed{8\vec{i} + 8\vec{j}}$$

$$5\vec{a} - 2\vec{b} = 5(2\vec{i} + \vec{j}) - 2(6\vec{i} + 7\vec{j})$$

$$= (10\vec{i} + 5\vec{j}) - (12\vec{i} + 14\vec{j})$$

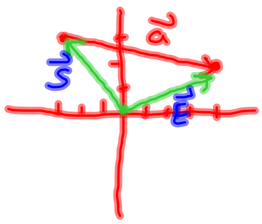
$$= \boxed{-2\vec{i} - 9\vec{j}}$$

$$|\vec{b}| = \sqrt{6^2 + 7^2} = \sqrt{85}$$

$$\vec{u}_{\vec{b}} = \frac{1}{\sqrt{85}}(6\vec{i} + 7\vec{j}) = \boxed{\frac{6}{\sqrt{85}}\vec{i} + \frac{7}{\sqrt{85}}\vec{j}}$$

Bonus Example:

Find a unit vector in the direction of the vector from $(-3,3)$ to $(4,2)$.



$\vec{S} = -3\vec{i} + 3\vec{j}$

$\vec{E} = 4\vec{i} + 2\vec{j}$

$$\vec{a} = \vec{E} - \vec{S} = (4\vec{i} + 2\vec{j}) + (+3\vec{i} - 3\vec{j})$$

$$= (4 + 3)\vec{i} + (2 - 3)\vec{j}$$

$$= 7\vec{i} - 1\vec{j}$$

$$|\vec{a}| = \sqrt{7^2 + (-1)^2} = \sqrt{50}$$

$$\vec{u} = \frac{1}{\sqrt{50}}(7\vec{i} - 1\vec{j})$$

$$= \boxed{\frac{7}{\sqrt{50}}\vec{i} - \frac{1}{\sqrt{50}}\vec{j}}$$