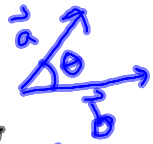
1 1.2: Dot Product



Definitions:

The dot product of the vectors **a** and **b** is given by $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Scalar

Dot Product computation formula If $\vec{a} = a_1\vec{i} + a_2\vec{j}$ and $\vec{b} = b_1\vec{i} + b_2\vec{j}$ then $\boxed{\vec{a}\cdot\vec{b} = a_1b_1 + a_2b_2}$

From the definition, it follows that the angle between two vectors is given by

If $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$

then $\boxed{\theta = \cos^{-1}\left(\dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right)}$ Use computation formula

a and **b** are orthogonal ($\perp$) if and only if $\vec{a}\cdot\vec{b} = 0$

Orthogonal complements (in 2-D only)

If $\vec{a} = a_1\vec{i} + a_2\vec{j}$, then

$\vec{a}^{\perp} = -a_2\vec{i} + a_1\vec{j}$

NOTE: $\vec{a}\cdot\vec{a}^{\perp}$

$= (a_1)(-a_2) + (a_2)(a_1)$

$= 0$

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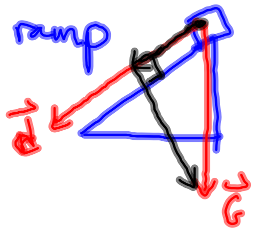
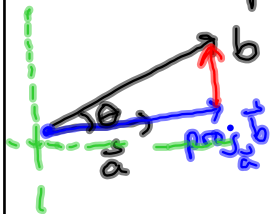
Scalar and Vector projections

Idea: Block on a ramp



General: project $\vec b$ onto $\vec a$



$$|\text{proj}_{\vec a}\vec b| = |\vec b|\cos\theta$$

$$|\text{proj}_{\vec a}\vec b| = \cancel{|\vec b|}\cdot\frac{\vec a\cdot\vec b}{|\vec a|\cancel{|\vec b|}}$$

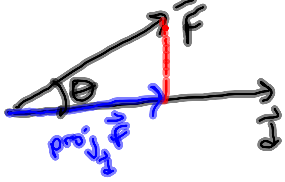
$$\boxed{\text{comp}_{\vec a}\vec b = \frac{\vec a\cdot\vec b}{|\vec a|}}$$

Scalar Projection of $\vec b$ onto $\vec a$

$$\boxed{\begin{aligned}\text{proj}_{\vec a}\vec b &= \frac{\vec a\cdot\vec b}{|\vec a|}\cdot\frac{1}{|\vec a|}\cdot\vec a\\ &= \frac{\vec a\cdot\vec b}{|\vec a|^2}\,\vec a\end{aligned}}$$

Vector Projection of $\vec b$ onto $\vec a$

Work A force exerted over a displacement



$$W = (|\vec F|\cos\theta)\cdot|\vec d|$$

$$\boxed{W = |\vec F|\,|\vec d|\cos\theta}$$

$$\boxed{W = \vec F\cdot\vec d}$$

$$W = \frac{\vec F\cdot\vec d}{\cancel{|\vec d|}}\cdot\cancel{|\vec d|}$$

Examples: $4\vec{i}+2\vec{j}$

If $\mathbf{a} = <4, 2>$ and $\mathbf{b} = \mathbf{i} + \mathbf{j}$, find the scalar and vector projection of $\mathbf{b}$ onto $\mathbf{a}$.

Scalar: $\text{comp}_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \dfrac{(4)(1)+(2)(1)}{\sqrt{4^2+2^2}} = \boxed{\dfrac{6}{\sqrt{20}}}$

Vector: $\text{proj}_{\vec{a}}\vec{b} = \dfrac{6}{\sqrt{20}} \cdot \dfrac{1}{\sqrt{20}} \cdot (4\vec{i}+2\vec{j})$

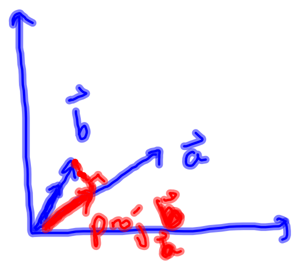
$= \dfrac{6}{20}(4\vec{i}+2\vec{j})$

$= \boxed{\dfrac{24}{20}\vec{i} + \dfrac{12}{20}\vec{j}}$

OR

$\text{proj}_{\vec{a}}\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\vec{a}$

$= \dfrac{6}{20}(4\vec{i}+2\vec{j})$



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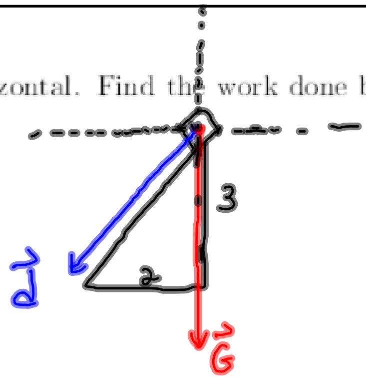
A 10 kg block slides down a ramp which is 3 m tall and 2 m horizontal. Find the work done by gravity if the block slides (friction-free) all the way down the ramp.



$\vec{G} = 0\vec{i} - \overset{(mg)}{98}\vec{j}$

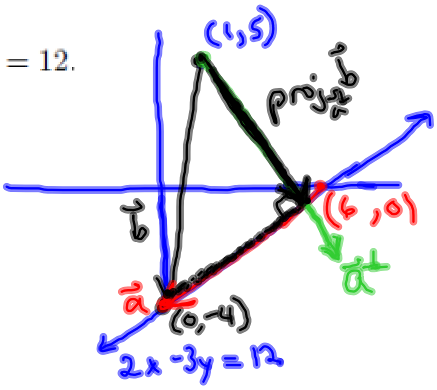
$\vec{d} = -2\vec{i} - 3\vec{j}$

$W = \vec{G} \cdot \vec{d}$

$= (0)(-2) + (-98)(-3)$

$= \boxed{294 \text{ N}\cdot\text{m} \text{ or } 294 \text{ J}}$

Find the distance from the point $(1, 5)$ to the line $2x - 3y = 12$.



① Find a vector from point to line

End-Start

$\vec{b} = (0\vec{i} - 4\vec{j}) - (\vec{i} + 5\vec{j})$

$= -\vec{i} - 9\vec{j}$

② Find a vector in the direction of the line

$\vec{a} = (0\vec{i} - 4\vec{j}) - (6\vec{i} + 0\vec{j})$

$= -6\vec{i} - 4\vec{j}$

$\vec{a}^\perp = 4\vec{i} - 6\vec{j}$

③ $\text{dist} = \left| \text{comp}_{\vec{a}^\perp} \vec{b} \right|$ $\quad \vec{b} = -\vec{i} - 9\vec{j}$, $\vec{a}^\perp = 4\vec{i} - 6\vec{j}$

$$= \frac{\vec{a}^\perp \cdot \vec{b}}{|\vec{a}^\perp|} = \frac{(4)(-1) + (-6)(-9)}{\sqrt{4^2 + (-6)^2}} = \boxed{\frac{50}{\sqrt{52}}}$$

$\vec{i}\ \ \vec{j}$

On Your Own: Given $\mathbf{a} =< 1, -1 >$ and $\mathbf{b} = \mathbf{i} + 2\mathbf{j}$, find $\mathbf{a} \cdot \mathbf{b}$ and the cosine of the angle between the vectors.

$$\vec{a} \cdot \vec{b} = (1)(1) + (-1)(2) = \boxed{-1}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{-1}{\sqrt{1^2+(-1)^2} \cdot \sqrt{1^2+2^2}}$$

$$= \frac{-1}{\sqrt{2}\sqrt{5}} = \boxed{\frac{-1}{\sqrt{10}}}$$

Find x such that $\mathbf{a} = < 4x - 5,\, x >$ is orthogonal to $\mathbf{b} = \mathbf{i} + x\mathbf{j}$.

Given $|\mathbf{a}| = 2$, $\mathbf{b}$ is a unit vector orthogonal to $\mathbf{a}$, and $\mathbf{c} = \mathbf{a} + \mathbf{b}$, find $\mathbf{a} \cdot \mathbf{c}$ and $\mathbf{b} \cdot \mathbf{c}$.