1 1.3: Vector Functions and Parametrized Curves

Definitions:

(Recall) function: A rule that assigns to each input a unique output

$$f: \mathbb{R} \to \mathbb{R}$$

Vector Valued function: input still real number
output is (2-D) vector

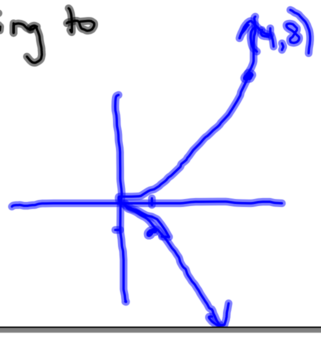
$$\vec{r}: \mathbb{R} \to \mathbb{R}^2$$

(Graph of vector function)
Parametrized Curve: set of all points corresponding to output vectors of my function

$\vec{r}(t) = t^2\vec{i} + t^3\vec{j}$

$\vec{r}(-1) = (-1)^2\vec{i} + (-1)^3\vec{j} = \vec{i} - \vec{j}$ corresponds to $(1,-1)$

$\vec{r}(2) = 4\vec{i} + 8\vec{j}$ $(4,8)$

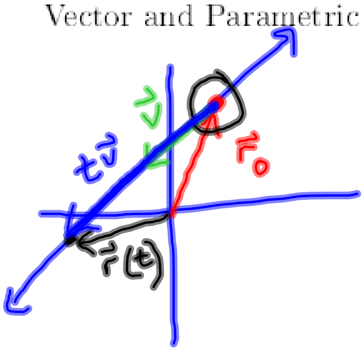


Eliminating the Parameter t $\quad x = \vec{i}$ component, $y = \vec{j}$ component (functions of t)

Writing an equation of the curve in x and y only

1) If possible, solve one equation for t
2) Substitute into other equation
3) (Trig) Identities are key (writing one of the functions in terms of other)

Vector and Parametric Equations of a Line



$\vec{r}_0$ = a vector corresponding to a point on the line

$\vec{v}$ = a vector in the direction of (parallel to) the line

$$\boxed{\vec{r}(t) = \vec{r}_0 + t\vec{v}}$$ Vector Equation of a Line

$y = b + xm$

if $\vec{r}_0 = x_0\vec{i} + y_0\vec{j}$ and $\vec{v} = a\vec{i} + b\vec{j}$, then

$$\vec{r}(t) = (x_0\vec{i} + y_0\vec{j}) + t(a\vec{i} + b\vec{j})$$
$$= \boxed{(x_0 + at)\vec{i} + (y_0 + bt)\vec{j}}$$

$$\boxed{\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \end{aligned}}$$ Parametric Equations

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Examples:

Given $\mathbf{r}(t) = \sin t\mathbf{i} + \cos^2 t\mathbf{j}$, eliminate the parameter to find the Cartesian equation of the curve. Is the point $(2, -3)$ on the curve?

$x = \sin t$

$y = \cos^2 t$

Identity $\sin^2 t + \cos^2 t = 1$

$x = \sin t$ so $x^2 = \sin^2 t$

$x^2 + y = 1$

$\boxed{y = 1 - x^2;\ -1 \le x \le 1}$ Parabola

$-3 = 1 - 2^2$ ✓

Better way to check $(2,-3)$:

$x = 2 \quad y = -3$

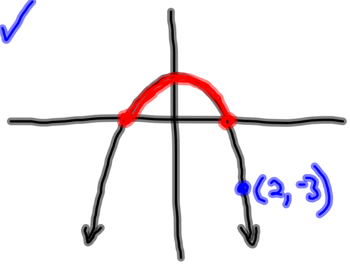
$\sin t = 2 \quad \cos^2 t = -3$

NO SOLUTION

Problem

$-1 \le \sin t \le 1$

$-1 \le x \le 1$



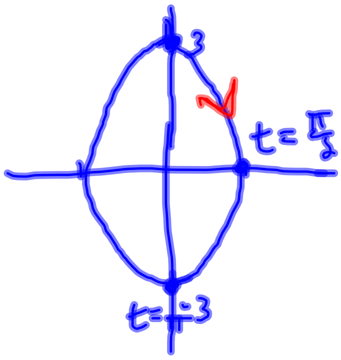
Describe the motion of a particle with position $\mathbf{r}(t) = < \overset{x}{2\sin t}, \overset{y}{3\cos t} >$, $\underset{\text{Start}}{0} \le t \le \underset{\text{End}}{2\pi}$.

Start $t=0$

$x = 2\sin 0 = 0$
$y = 3\cos 0 = 3$
$(0,3)$

End $t = 2\pi$

$x = 2\sin 2\pi = 0$
$y = 3\cos 2\pi = 3$
$(0,3)$



Path (Eliminate Parameter)

$\sin^2 t + \cos^2 t = 1$

$x = 2\sin t$
$\frac{x}{2} = \sin t$

$\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$

$\boxed{\frac{x^2}{4} + \frac{y^2}{9} = 1}$ Ellipse

Direction (if needed)

Clockwise

$t = \frac{\pi}{2}$

$x = 2\sin\frac{\pi}{2} = 2$
$y = 3\cos\frac{\pi}{2} = 0$
$(2,0)$

Find vector and parametric equations for the line passing through the points $(-3, 4)$ and $(2, 8)$.

$$\vec{r}(t) = \vec{r}_0 + t\vec{v}$$

$\vec{r}_0 = -3\vec{i} + 4\vec{j}$ (can also be $2\vec{i} + 8\vec{j}$)

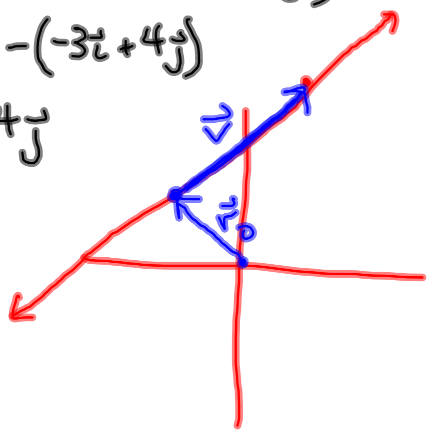
$$\vec{v} = (2\vec{i} + 8\vec{j}) - (-3\vec{i} + 4\vec{j}) = 5\vec{i} + 4\vec{j}$$

$$\boxed{\vec{r}(t) = (-3\vec{i} + 4\vec{j}) + t(5\vec{i} + 4\vec{j})}$$

Vector

$$\vec{r}(t) = \boxed{\begin{aligned} &(-3 + 5t)\vec{i} + (4 + 4t)\vec{j} \\ &x = -3 + 5t \\ &y = 4 + 4t \end{aligned}}$$

Parametric



On Your Own: Given the curve parametrized by $\mathbf{r}(t) = (\overset{x}{t^2 + 1})\mathbf{i} + (\overset{y}{t^2 - 1})\mathbf{j}$, determine when, if at all, the curve passes through the point $(\overset{x}{5}, \overset{y}{3})$

Need to solve $\quad t^2+1=5 \qquad t^2-1=3$

$$t^2=4 \qquad t^2=4$$

$$t=\pm 2 \qquad t=\pm 2$$

Solutions to <u>BOTH</u> equations

$$\boxed{t=2 \text{ or } t=-2}$$