1 2.3: Analytic Computation of Limits

Properties of Limits: (pp 91-93. Basis for the techniques used in the following examples.)

Examples: (Using Properties)

$$\lim_{x\to 1} x^3 - 3x^2 + 1 = \lim_{x\to 1} x^3 - \lim_{x\to 1} 3x^2 + \lim_{x\to 1} 1$$

$$= \lim_{x\to 1} x^3 - 3\lim_{x\to 1} x^2 + \lim_{x\to 1} 1$$

$$= \left(\lim_{x\to 1} x\right)^3 - 3\left(\lim_{x\to 1} x\right)^2 + \lim_{x\to 1} 1$$

$$= 1^3 - 3(1)^2 + 1$$

evaluated at $x=1$

$$= 1 - 3 + 1 = \boxed{-1}$$

$$\lim_{y\to 2}\frac{\sqrt{y+5}-\sqrt{7}}{y-2}\circ\frac{(\sqrt{y+5}+\sqrt{7})}{(\sqrt{y+5}+\sqrt{7})}$$

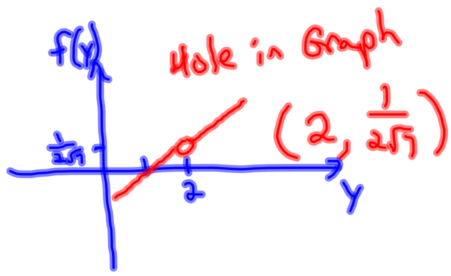
Rationalize

$$=\lim_{y\to 2}\frac{(y+5)-7}{(y-2)(\sqrt{y+5}+\sqrt{7})}$$

$$=\lim_{y\to 2}\frac{\cancel{y-2}}{\cancel{(y-2)}(\sqrt{y+5}+\sqrt{7})}$$

$$=\frac{1}{\sqrt{2+5}+\sqrt{7}}=\boxed{\frac{1}{2\sqrt{7}}}$$

What is happening?



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$$\lim_{t \to a} \left(x(t)\vec{i} + y(t)\vec{j} \right) = \left(\lim_{t \to a} x(t) \right)\vec{i} + \left(\lim_{t \to a} y(t) \right)\vec{j}$$

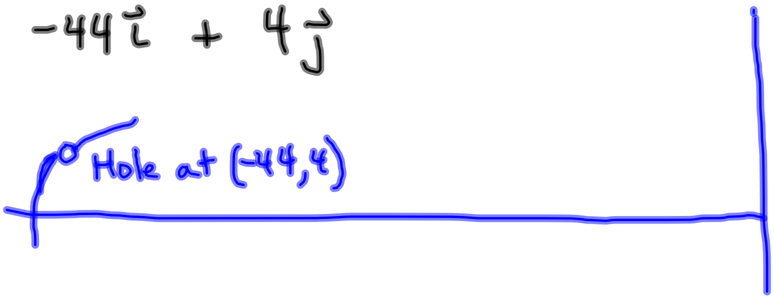
"Limit of Each Component"

$\lim\limits_{t \to 2} \mathbf{r}(t)$ where $\mathbf{r}(t) = \left(\frac{5t^3 + 4}{t - 3} \right) \mathbf{i} + \left(\frac{t^2 - 4}{t - 2} \right) \mathbf{j}$

$$= \left(\lim_{t \to 2} \frac{5t^3+4}{t-3} \right)\vec{i} + \left(\lim_{t \to 2} \frac{t^2-4}{t-2} \right)\vec{j}$$

$$= \frac{5(2)^3+4}{2-3}\vec{i} + \left(\lim_{t \to 2} \frac{(t+2)\cancel{(t-2)}}{\cancel{t-2}} \right)\vec{j}$$

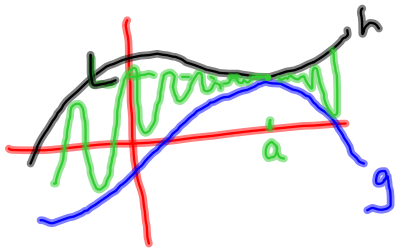
$$= -44\vec{i} + 4\vec{j}$$



Squeeze Theorem: If $g(x) \le f(x) \le h(x)$ and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$

$f(x)$

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) =$$

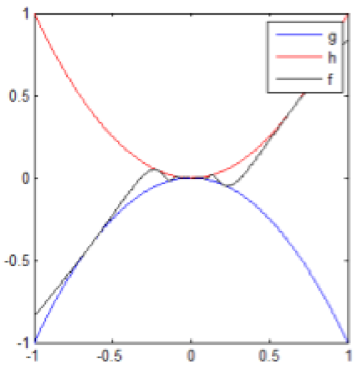


$$x^2(-1) \le x^2 \sin\left(\frac{1}{x}\right) \le 1(x^2)$$

$$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

$$\lim_{x\to 0} -x^2 = 0$$

$$\lim_{x\to 0} x^2 = 0$$



$\therefore \lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$ by Sqz Thm

On Your Own: $\lim_{x \to -4} \frac{2x+8}{x^2+x-12} = \frac{0}{0}$

$$= \lim_{x \to -4} \frac{2\cancel{(x+4)}}{\cancel{(x+4)}(x-3)}$$

$$= \frac{2}{-4-3} = \boxed{\frac{-2}{7}}$$

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