1 3.11: Linear and Quadratic Approximation

Purpose: To understand Linear (Differential) and Quadratic Approximation to a function near a certain point.

Recall: Given $y = f(x)$, the tangent line at $x = a$ is the best approximation to the graph of f "near" $x = a$.

Why?

Formula for the tangent line:

Therefore, if we want to approximate values of f near a given x-value (a), we can use the tangent line to obtain these approximations.

Example:

Use the liner approximation at $x = \dfrac{27}{8}$ of an appropriate function to estimate $\sqrt[3]{3}$.

A different view: Because of the Linear Approximation, for values of x "near" $x = a$, we have

$$f(x) \approx L(x) = f(a) + f'(a)(x - a)$$

Examples

The circumference around the middle of a sphere is measured to be 40cm, with a possible error of ± 1 cm. Use differentials to estimate the possible error in the volume of the sphere.

Quadratic Approximation:

(On your own): Use differentials or linear approximation to approximate $\sin(33^\circ)$

$$\frac{1}{2} + \frac{\sqrt{3}\,\pi}{120} \approx 0.5453$$