1 3.3: Rates of Change

Recall: The derivative of a function can measure:

1) slope of tangent line

2) velocity if f = position

3) rate of change

Examples: $64-96+36$

A particle moves in a line according to the function $s = f(t) = t^3 - 6t^2 + 9t$, where t is in seconds and s is in feet.

a) Find the velocity at time t

b) What is the velocity after 2 seconds?

c) When is the particle at rest? $v=0$

d) Find the total distance traveled in the first 4 seconds.

a) $f'(t) = 3t^2 - 12t + 9$

b) $f'(2) = 3(2)^2 - 12(2) + 9 = 12 - 24 + 9 = \boxed{-3 \ \frac{ft}{s}}$

c) $f'(t) = 3t^2 - 12t + 9 = 0$

$3(t^2 - 4t + 3) = 0$

$3(t-3)(t-1) = 0$

$\boxed{t=3 \quad t=1 \quad \text{seconds}}$

d) dist $\neq f(4) - f(0)$ (displacement)

$f(1) - f(0) = 4 - 0 = 4$ ft

$d = |f(3) - f(1)| = |0 - 4| = 4$ ft

$d = f(4) - f(3) = 4 - 0 = 4$ ft

Total $= \boxed{12 \text{ ft}}$

Sand is dumped into a cylindrical can with a 3 foot diameter.

a) Find the average rate of change in the volume of the sand from a height of 1 foot to a height of 2 feet.

m_{sec}

b) Find the instantaneous rate of change in the volume of sand when the height is 2 feet.

deriv

a) $V = \pi r^2 h \qquad r = \frac{3}{2}$

$V = \pi \left(\frac{3}{2}\right)^2 h = \frac{9\pi}{4} h \qquad \frac{\Delta V}{\Delta h} \quad \frac{V(2) - V(1)}{2-1} = \frac{2 \cdot \frac{9\pi}{2 \cdot 2} - \frac{9\pi}{4}}{2-1} = \frac{9\pi}{4} \frac{ft^3}{ft}$

b) $\frac{dV}{dh} = \boxed{\frac{9\pi}{4} \frac{ft^3}{ft}}$

Linear = Const ROC