

1 3.4: Derivatives of Trig Functions

Key Limit: $\lim_{x \rightarrow 0} \frac{\sin x}{x} =$

"Proof": (zoom in on graph of $y = \sin x$ at $x = 0$)

Key Limit: $\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} =$

Proof:

We can use these limits to find the derivative of $f(x) = \sin x$ using the definition:

Similarly, we can show that $\frac{d}{dx}(\cos x) = -\sin x$. Once we know these, we can find the derivative of all the other trig functions using identities if needed.

Example: $\frac{d}{dx}(\tan x) = \sec^2 x$

Other derivatives:

Examples:

Compute $\lim_{x \rightarrow 0} \frac{\sin 5x}{\tan 3x}$

(On Your Own): You already know the identity $\sin(2x) = 2 \sin x \cos x$. What do you obtain when you differentiate the right hand side of this identity?

$2 \cos 2x$

Find all values of a such that $0 \leq a \leq 2\pi$ and the line tangent to $f(x) = \frac{\cos x}{2 + \sin x}$ at $x = a$ is horizontal.

$$\left(x = \frac{7\pi}{6}, x = \frac{11\pi}{6}\right)$$