1 3.4: Derivatives of Trig Functions

Key Limit: \( \lim_{x \to 0} \frac{\sin x}{x} = \)

"Proof": (zoom in on graph of \( y = \sin x \) at \( x = 0 \))

Key Limit: \( \lim_{x \to 0} \frac{\cos x - 1}{x} = \)

Proof:

We can use these limits to find the derivative of \( f(x) = \sin x \) using the definition:
Similarly, we can show that \( \frac{d}{dx}(\cos x) = \) Once we know these, we can find the derivative of all the other trig functions using identities if needed.

Example: \( \frac{d}{dx}(\tan x) = \) 

Other derivatives:
Examples:

Compute \( \lim_{x \to 0} \frac{\sin 5x}{\tan 3x} \)

(On Your Own): You already know the identity \( \sin(2x) = 2 \sin x \cos x \). What do you obtain when you differentiate the right hand side of this identity?

\[ 2 \cos 2x \]

Find all values of \( a \) such that \( 0 \leq a \leq 2\pi \) and the line tangent to \( f(x) = \frac{\cos x}{2 + \sin x} \) at \( x = a \) is horizontal.

\[ (x = \frac{7\pi}{6}, \quad x = \frac{11\pi}{6}) \]