# 1 3.5: Chain Rule

$\frac{d}{dx}(2\sin x \cos x) = 2\cos 2x$

From 3.4: we know $\frac{d}{dx}(\sin x) = \cos x$. Does $\frac{d}{dx}(\sin 2x) = \cos 2x$? NO

**Recall**: The **composition** of 2 functions $f$ and $g$ is defined by

$(f \circ g)(x) = f(\underbrace{g(x)}_{\text{inside}})$

Define $f$ and $g$ for the above function.

$\sin(2x)$

$f(x) = \sin x$

$g(x) = 2x$ NOTE $g'(x) = 2$

Oct 5-12:31 PM

$f(g(x))$

**The Chain Rule**: If $f$ and $g$ are differentiable functions, $y = f(u)$ and $u = g(x)$, then

$$\frac{dy}{dx} = \boxed{\frac{dy}{du} \cdot \frac{du}{dx}}$$

$y = \sin u$
$u = 2x$

$\frac{dy}{du} \cdot \frac{du}{dx}$
$= \cos u \cdot 2 = 2\cos 2x$

An alternate version of the Chain Rule states that $\frac{d}{dx} f(g(x)) =$

$$\frac{dy}{du} \cdot \frac{du}{dx}$$
$$f'(u) \cdot g'(x)$$
$$f'(g(x)) \cdot g'(x)$$

$$\boxed{f'(g(x)) \cdot g'(x)}$$

$u/g(x)/$ stuff

$u/g(x)/$ stuff

Find the derivatives of the following:

$f(x) = \sin(x^2) - \sin^3 x$ $\quad (\sin x)^3$

$$f'(x) = \boxed{\cos(x^2) \cdot 2x - 3\sin^2 x \cdot \cos x}$$

$$y = \sqrt{\cos^2(3x)+1} = \left(\cos^2(3x)+1\right)^{1/2}$$

$$= \left(\underbrace{(\cos(3x))^2+1}_{\text{stuff}}\right)^{1/2}$$

Multiple composition → Outside/In

$$y' = \frac{1}{2}\left((\cos(3x))^2+1\right)^{-1/2}\cdot\frac{d}{dx}\left(\underbrace{(\cos(3x))}_{\text{more stuff}}{}^2+1\right)$$

$$= \frac{1}{2}\left((\cos(3x))^2+1\right)^{-1/2}\cdot(2)(\cos(3x))\cdot\frac{d}{dx}\left(\cos(\underbrace{3x}_{\text{even more stuff}})\right)$$

$$= \frac{1}{2}\left((\cos(3x))^2+1\right)^{-1/2}\cdot 2(\cos(3x))\cdot(-\sin(3x))\cdot\frac{d}{dx}(3x)$$

$$= \boxed{\frac{1}{2}\left((\cos(3x))^2+1\right)^{-1/2}\cdot 2(\cos(3x))\cdot(-\sin(3x))\cdot 3}$$

Differentiate the following:

$f(x) = x^2 \tan(3x)$

Product Rule

$$f'(x) = \boxed{x^2 \cdot (\sec^2(3x)) \cdot 3 + \tan(3x) \cdot 2x}$$

$1^{st} \cdot d(2^{nd}) + 2^{nd} \cdot d(1^{st})$

$$= \boxed{3x^2 \sec^2(3x) + 2x\tan(3x)}$$

Oct 5-12:32 PM

$(\sin x)^{-2}$

Quotient Rule OR $(2x+1)\csc^2 x$

$$y = \frac{2x+1}{\sin^2 x} \quad \begin{matrix}\text{HI} \\ \text{LO}\end{matrix}$$

$(\sin x)^2$

$$y' = \boxed{\frac{(\sin^{\cancel{2}} x)(2) - (2x+1)\cdot 2\cancel{\sin} x \cdot \cos x}{\sin^{3} x}}$$

OR

$$= \boxed{\frac{2\sin x - 2(2x+1)\cos x}{\sin^3 x}}$$

(On Your Own) Given $f(1) = 2$, $f(2) = 2\sqrt{2}$, $f'(1) = 1$, $f'(2) = \frac{1}{2}$, $g(1) = 2$, $g(2) = \frac{5}{2}$, $g'(1) = \frac{3}{4}$, $g'(2) = 0$, and $u(x) = f(g(x))$, find $u'(1)$

$$u'(x) = f'(g(x))\, g'(x)$$

$$u'(1) = f'(g(1))\, g'(1)$$

$$= f'(2) \cdot \frac{3}{4}$$

$$= \frac{1}{2} \cdot \frac{3}{4}$$

$$= \boxed{\frac{3}{8}}$$

Oct 5-12:32 PM