1 3.6: Implicit Differentiation

The equation $F(x, y) = 0$ **implicitly** defines a relation (not necessarily a function) between y and x. The **graph** of $F(x, y) = 0$ is the set of all points (x, y) such that the equation holds ($\{(x, y)|F(x, y) = 0\}$) Given a graph of an implictly-defined relation, we can still talk about the slope of the line tangent to the curve at the given point.

Method for Implicit Differentiation:

1. Done when y is not explicitly defined as a function of x.
2. Differentiate both sides of the equation, remembering that y depends on x (you can call it $y(x)$ if that helps)
3. Solve for $y'(x)$ or $\frac{dy}{dx}$.

Examples:

Find $\frac{dy}{dx}$ if $y^5 + 3x^2y^2 + 5x^4 = 12$.

Find the equation of the line tangent to $x^2 + xy + y^2 = 7$ at the point $(1, -3)$.

Show that the curves $y = 3x^2$ and $x^2 + 2y^2 = 19$ are orthogonal.

(On your own): The equations $y = mx$ and $x^2 + y^2 = r^2$ represent **families of curves** for different constants r and m. Show that these families of curves are orthogonal (for all values of the constants)

(Slopes are $\frac{y}{x}$ and $-\frac{x}{y}$ which are negative reciprocals.)