1 3.6: Implicit Differentiation

The equation \( F(x, y) = 0 \) implicitly defines a relation (not necessarily a function) between \( y \) and \( x \). The graph of \( F(x, y) = 0 \) is the set of all points \((x, y)\) such that the equation holds (\( \{(x, y) | F(x, y) = 0\} \)). Given a graph of an implicitly-defined relation, we can still talk about the slope of the line tangent to the curve at the given point.

Method for Implicit Differentiation:

1. Done when \( y \) is not explicitly defined as a function of \( x \).
2. Differentiate both sides of the equation, remembering that \( y \) depends on \( x \) (you can call it \( y(x) \) if that helps)
3. Solve for \( y'(x) \) or \( \frac{dy}{dx} \).

Examples:

Find \( \frac{dy}{dx} \) if \( y^2 + 3x^2y^2 + 5x^4 = 12 \).

Find the equation of the line tangent to \( x^2 + xy + y^2 = 7 \) at the point \((1, -3)\).
Show that the curves $y = 3x^2$ and $x^2 + 2y^2 = 19$ are orthogonal.

(On your own): The equations $y = mx$ and $x^2 + y^2 = r^2$ represent families of curves for different constants $r$ and $m$. Show that these families of curves are orthogonal (for all values of the constants).

(Slopes are $\frac{y}{x}$ and $-\frac{x}{y}$ which are negative reciprocals.)